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LAMINAR-TURBULENT TRANSITION IN CO-CURRENT
STRATIFIED OIL-WATER FLOW

BY

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE

IN

CHEMICAL ENGINEERING

FACULTY OF ENGINEERING

DEPARTMENT OF CHEMICAL AND PETROLEUM ENGINEERING

EDMONTON, ALBERTA

APRIL, 1967

UNIVERSITY OF ALBERTA
FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled LAMINAR-TURBULENT TRANSITION IN CO-CURRENT STRATIFIED OIL-WATER FLOW, submitted by Harry S. Stellmach in partial fulfillment of the requirements for the degree of Master of Science in Chemical Engineering.

ABSTRACT

An experimental investigation was conducted to characterize the laminar-turbulent transition of water in co-current, stratified flow with oil. Experiments were performed in a horizontal rectangular conduit having an overall length of 37.5 feet and an aspect ratio of 7.95:1. A hot-film anemometer and Dynograph recorder were used to detect the instantaneous velocity fluctuations in the water phase.

Laminar-turbulent transition measurements in water were obtained for water Reynolds numbers between 0 and 3200 and oil Reynolds numbers between 0 and 745.

Transition from laminar to turbulent flow was characterized by three phases: development of stable sinusoidal velocity fluctuations; formation of sharply defined spots of disturbed or turbulent flow; and growth of turbulent spots to occupy the entire flow field.

An intermittency number and intermittency factor were determined for each set of flow rates. For intermittency factors less than 0.5, the degree of turbulence increased due to the formation of additional spots. For intermittency factors greater than 0.5 increased turbulence was due to the expanding growth and melting together of the turbulent spots. The data suggests a normal Gaussian distribution for the intermittency factor with respect to the water Reynolds number.

Three regions, defined by different oil Reynolds numbers, indicate the influence of the oil layer upon the laminar-turbulent transition in the water. For oil Reynolds numbers less than 35, the oil layer appears to stabilize the water phase. For oil Reynolds numbers between 35 and 330, the oil layer seems to have little or no effect on the water phase. For oil Reynolds numbers greater than 330, a strong influence upon the laminar-turbulent transition in the water phase is suggested by a sudden increase in the transition water Reynolds number. The fluctuating velocity component frequencies and the intermittent flow patterns were also distinctly different for Region 3 than for Regions 1 and 2.

Interfacial disturbances coinciding with intermittent turbulent spots were noted. These disturbances were very weak at low oil Reynolds numbers but developed into small two-dimensional waves for oil Reynolds numbers greater than 300.

An attempt was made in this investigation to analyse theoretically the laminar-turbulent transition in the two-phase system. Predicted critical transition Reynolds numbers of water for oil Reynolds numbers less than 330, and of oil for water Reynolds numbers less than 1000, were in good agreement with empirical values.

ACKNOWLEDGEMENT

The author wishes to express his sincere gratitude to Dr. L.U. Lilleleht and Dr. J.T. Ryan for their guidance and encouragement throughout the course of this investigation.

Thanks are also extended to the National Research Council for financial aid during the project.

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I. INTRODUCTION

Multiphase flow systems are commonly encountered in industry. Practical examples are pipelines, tubular reactors, film coolers, and absorbers. Flow instabilities can greatly influence the flow field and transport phenomena. Considerable research has been done to determine the physical mechanisms and types of instabilities responsible for the transition from one flow regime to another.

The present study was initially prompted by the findings of an earlier experimental investigation by Charles(4,5) on the stability of co-current, stratified flow of oil and water in a horizontal rectangular conduit. Using pressure measurements he established the transition range of each of the fluids in single-phase flow to be between the Reynolds numbers of 2300 and 3500. He then determined the transition flow rate of the water at various oil flow rates and the transition flow rate of the oil for various water rates. For two-phase flow he established the laminar-turbulent transition by observing the breakup of dye filaments injected into both phases. Charles' results are illustrated in Figure 1.

It appears that in the two-phase system the laminar oil layer has a stabilizing effect on the water layer. That is, the transition to turbulence in the water phase occurs at a higher Reynolds number in the presence of a thin laminar

oil layer than with no oil present. Alternately, an already turbulent water layer tends to bring about transition in the oil phase at a lower Reynolds number than if the oil alone were present in the channel. Similar observations were made by Gazley(14) with an air-water system. This phenomenon of the delay of transition to the turbulent flow condition is of interest and potential importance to many engineering problems.

The purpose of this work was to experimentally determine the effect of a laminar oil layer on the transition to turbulence of the water phase. The turbulence structure within the single- and two-phase liquid system for laminar-turbulent transition was examined experimentally by studying the intermittent formation of turbulent patches in the less viscous water phase. The characteristic fluid motion within the water phase was detected by means of a hot-film anemometer. An attempt was also made to theoretically analyze the experimental data.

II. LITERATURE REVIEW

A. General Concepts of Laminar, Transitional, and Turbulent Flow

One of the fundamental problems of fluid mechanics is concerned with the onset of turbulence. Consequently, much research has been done on the transitional flow regime in which neither completely laminar nor completely turbulent flow exists.

The accepted criterion for the onset of turbulence is the dimensionless Reynolds number $U_b d / \nu$. The critical or transition Reynolds number is the magnitude of this group at which transition to turbulence occurs. Its value depends on the conduit geometry and the characteristic length d . The theoretical and empirical prediction of the critical Reynolds number for various systems is important.

Earlier experimental investigations(8) have revealed that transition to turbulence is not instantaneous. The flow becomes intermittent in a region around the critical Reynolds number. Intermittent flow is characterized by the appearance of turbulent spots within the laminar stream. It implies the presence of definite interfaces separating regions of laminar and disturbed flow. The spots originate(32) at the conduit entry section and at the boundary as well as within the flow field. Once these spots have been created they grow laterally and axially as they are swept along

with the fluid until finally the entire stream is in turbulent motion(10,11). The cycling between an apparently laminar and disturbed flow can be described by an intermittency factor γ , which represents the fraction of time the flow at a fixed point is turbulent. Thus, $\gamma = 0$ represents fully laminar flow and $\gamma = 1$ a fully turbulent motion.

The phenomenon of intermittency was first observed by Corrsin(8) in 1943 and was recognized as one of the important features of the transitional flow regime. Since then it has been encountered under many different experimental conditions. To date this concept has been incorporated only to a limited extent into the theoretical aspects of research on turbulence.

Rotta(29) in his experiments with air and water found that for pipe flow the alternately laminar-turbulent motion occurred in a range of Reynolds numbers between 2200 and 2600. Values of the intermittency factor γ for air were obtained by recording the velocity fluctuations with a hot-wire anemometer. His method for measuring γ in the water was based on the difference in the mean momentum for turbulent and laminar motion at constant mass flow rate. The water in the transition flow regime was allowed to discharge freely from a pipe. The fluid trajectory fluctuated between two limiting positions. The amount of water collected at each position was proportional to the time the flow field at the

discharge point was in laminar or turbulent flow. The number of changes from one flow pattern to the other were equal to the number of turbulent patches. Using the above techniques Rotta studied the effect of different entrance nozzles and entrance lengths on the relationship between intermittency and Reynolds number.

In a transition boundary layer flow, turbulent spots are created at some point and pass downstream with a characteristic velocity, shape and growth rate. Emmons(12) formulated a theory to describe the boundary layer transition process based on probability concepts. His theory assumes a turbulent spot production function which can be related to the probability of the flow being turbulent at any one point. He assumed:

1. point-like breakdown;
2. a sharp boundary between the turbulent spot and the surrounding laminar flow;
3. a uniform rate of spot growth;
4. no interaction between spots.

The validity of these assumptions was verified experimentally by Schubauer and Klebanoff(33) and later by Elder(11) who studied the form, growth and interaction of turbulent spots. Dhawan and Narasimha(10) further extended and substantiated Emmons' work. They also stressed the superposition principle of fully laminar and turbulent properties

by applying a weighting factor of $(1-\gamma)$ and γ respectively to obtain mean flow velocities for the transition regime. Dhawan and Narasimha pointed out that the γ value near the wall is the important characteristic property for the transition region. Variation of γ with respect to the distance from the wall has only a secondary influence in determining the mean transition velocity profiles. Both Dhawan and Narasimha and Elder indicated that the growth of turbulent spots was independent of the initiating agent. Dhawan and Narasimha suggested the probable existence of a relation between the rate of production of turbulent spots and the Reynolds number. Elder, however, suggests that breakdown to turbulence occurs at any point where the velocity fluctuations exceed a critical intensity.

A brief but comprehensive review of the intermittent nature of transition flow is given by Coles(7).

B. Hydrodynamic Stability Theory

The objects of any theory of hydrodynamic stability are:

1. to determine whether a given flow is unstable at a sufficiently large Reynolds number;
2. to predict the minimum Reynolds number at which a specific laminar flow system becomes unstable; and
3. to postulate the physical mechanisms involved in the transition process.

Early theories which were successful in predicting a critical Reynolds number of the correct order of magnitude were based on the method of small disturbances(20,32). They were formulated on the premise that the main flow is stable if a small disturbance is damped out and unstable if it is amplified. Thus, the breakdown of laminar flow can be studied by following the time behavior of a small disturbance superimposed on the mean flow. If the disturbances are small compared with the corresponding main flow values, only the linear terms of the velocity fluctuations need be considered. Both the disturbances as well as the main flow are governed by the Navier-Stokes and continuity equations.

Parallel flows have received the most attention from this stability theory. Squire(36) was able to show that three-dimensional wavy disturbances are more stable than two-dimensional ones. Hence, in order to simplify the problem, only the propagation of two-dimensional periodic disturbances is considered. The analysis leads to a fourth order differential equation.

Results of these stability calculations suggest that small disturbances in velocity within a comparatively narrow range of wavelengths and frequencies are amplified, whereas all other disturbances are damped. Breakdown of laminar flow is assumed to occur when the amplified disturbances have grown to a sufficiently large value.

This theory of selective damping and amplification appears to be substantiated by Schubauer and Skramstad(34) who found regular sinusoidal oscillations within the laminar boundary layer on a flat plate. These fluctuations were detected using a hot-wire anemometer in a wind tunnel having a very low level of turbulence. The amplitude of the oscillations increased as the hot-wire pickup was moved downstream along the flat plate. Bursts of very large amplitude oscillations developed as transition was approached. At the point of transition the flow broke into spots of irregular high-frequency fluctuations. The regular sinusoidal motion had escaped earlier detection because of the high level of background turbulence intensity in previous measurements. Under these conditions transition is not preceded by selective amplification of sinusoidal oscillations but is caused directly by the irregular disturbances.

Prengle and Rothfus(28) have also reported the presence of stable sinuous wave motion in water over a range of Reynolds numbers from 1225 to 2100. The phenomenon was observed in water by injecting thin dye filaments at various points of a circular pipe cross-section.

An attempt to predict theoretically the onset of periodic motion of finite amplitude in flow between parallel planes has been made by Meksyn and Stuart(24). Their calculations indicate an onset of oscillatory motion at a minimum

Reynolds number of 2900. The critical Reynolds number increased with decreasing disturbance amplitude. The stability calculations performed by Lin(20) on plane Poiseuille flow yielded a critical Reynolds number of 5314. Numerical calculations by Thomas(39) verified this result by obtaining a critical Reynolds number of 5780. The Reynolds number in the above three cases was based on the maximum velocity and the half separation of the plates. Thomas also confirmed Lin's calculations that for a disturbance of a given wave number, the flow is unstable for only a finite range of Reynolds numbers.

Linear theory indicates the complex role of viscosity in determining the stability of two-dimensional parallel flows. The effect of viscosity can be destabilizing at very large Reynolds numbers. A disturbance of finite wavelength which is damped at very large Reynolds numbers becomes amplified by the action of viscosity as the Reynolds number is decreased. A further decrease in Reynolds number again causes the disturbances to be damped.

Linear stability theory can give useful results about transition to turbulence. The theory is, however, limited to regions where non-linear effects are small. As transition is approached, non-linear effects become important and the usefulness of linear theory decreases. The papers of Stuart(38) and Watson(40) consider the two-dimensional, non-

linear problem of instability in plane Poiseuille flow. A method is indicated in which the Reynolds number is fixed and the decay or growth of a disturbance with time is followed.

The assumption of two-dimensional disturbances made by many authors, appears valid during the initial stages of disturbance growth (Squire(36)). However, as turbulence is characterized by random three-dimensional fluctuations, three-dimensional effects may become a controlling factor during some stage of turbulence development. Benney's work(2) on non-linear oscillations in parallel shear flow gives special attention to the three-dimensional oscillations. He investigated finite amplitude oscillations during laminar flow breakdown using a perturbation technique similar to linear theory, but retaining the second order term of the perturbation series. The analysis indicated that second order velocities, induced by the primary oscillations, form a cellular vortex structure. A periodic spanwise variation results, which produces a redistribution of momentum perpendicular to the flow direction. Momentum is extracted from the viscous region at one position and is reintroduced into that region at a lateral position. The secondary vortices are modified by the superposition of the vorticity of the primary oscillation itself, which is periodic in the downstream direction. The two structures interact and cause alternate cancellation and reinforcement of the oscillations. This momentum exchange is respons-

sible for the alternate steepening and flattening of the velocity profile during transition.

C. Instabilities in Two-Phase Flows

To date, much of the work on two-phase flow instability has been experimental. Attention has been directed mainly toward film break-up, interfacial disturbances and interfacial waviness. Existing theoretical stability analyses generally consider flow systems which are unbounded in the direction perpendicular to the surface. The problem at hand considers flow instability within a fluid phase and deals with a completely enclosed or internal system.

Experimental observations were made by Charles(4,5) for co-current flow of oil and water in a rectangular conduit. In the oil and water flow ranges investigated by Charles, five flow regimes of distinct interfacial structure were noted: smooth, small two-dimensional waves, large two-dimensional waves, roll waves, and three-dimensional waves. Photographs of these interfacial structures are given in references (4) and (5). At low flow rates, with both phases in laminar flow, the interface was completely smooth. Successive transitions occurred upon steadily increasing the water rate and maintaining the oil rate constant. Small, two-dimensional waves of low amplitude and small wavelengths having crests several times longer than the wavelength were the first to appear.

These were followed by large two-dimensional waves which had crests extending over the entire width of the channel with the original small waves still superimposed. Large two-dimensional waves became roll waves at still higher water rates. Roll waves had sharp irregular crests and variable wavelengths. With both phases in turbulent flow, the interfacial waves became three-dimensional having crest lengths approximately equal to the wavelength. These flow regimes and the corresponding interfacial structures are shown in Figure 1.

The above observations agree with those of Hanratty and Engen(18) who studied the interfacial structure of a two-phase air-water system as well as with those of Strashok(37) who used a photometric technique to characterize the interfacial disturbances of two-phase oil-water flow.

Cohen and Hanratty(6) studied the interfacial wave generation in co-current flow of air and a liquid within a rectangular conduit. Emphasis was placed on the transition from two-dimensional to three-dimensional wave regimes. It was observed that the air velocity at transition increased with an increase in liquid viscosity and a decrease in liquid height. Two-dimensional waves appeared to be more stable for the more viscous solutions. Wave velocities were found to be larger than the maximum liquid velocity, but smaller than the average gas velocity. Cohen and Hanratty's interpretation of the experimental and theoretical results was that the wavy liquid

film obtains its energy from the air flow. The interfacial waves cause disturbances in the air velocity field which result in shear stress and pressure variation at the liquid surface. Energy is transferred at the interface from the air to the liquid by shear stress fluctuations in phase with wave height and by pressure variations in phase with the wave slope. Shear stress fluctuations transmit energy to the liquid in the horizontal direction; pressure fluctuations transmit energy to the liquid in the vertical direction. Waves will decay if the rate of viscous dissipation is greater than the rate of energy transferred to the liquid film. The above mechanism is unlike that proposed by Phillips(27). The latter is concerned with the resonance effect of pressure fluctuations within an air stream on an initially calm water interface. Fluctuation occurring as a result of this mechanism should be of the same order of magnitude as the average air velocity.

The stability analysis of Feldman(13) considers two viscous fluids in parallel, plane motion. One fluid is bounded by a solid wall below and by the other fluid above; the other fluid is unbounded above. The fluids are in steady motion parallel to the interface and each fluid has a uniform rate of shear. The mathematical analysis is based on the small disturbance theory and leads to the inclusion of a number of parameters important for the non-homogeneous system. Similar to the single phase problem, the disturbance phase velocity

and wave number as well as the lower fluid Reynolds number are involved. In addition, the Froude number accounts for the effect of gravity and the Weber number accounts for the influence of the interfacial tension. The density and viscosity ratios are also important parameters. Results are presented graphically as neutral stability curves on plots of wave number versus Reynolds number and are similar to those for homogeneous fluids. Feldman found that varying the density ratio could be stabilizing or destabilizing whereas increasing the viscosity ratio always tended to stabilize the flow, provided the ratio was not small compared to unity. He also observed that surface tension effects are negligible when the disturbance frequency is small but become important at large frequencies.

Ostrach and Koestel(26) summarized the major types of interfacial instability and discussed the physical mechanisms which can alter the flow behavior in a two-phase system. A criterion for determining the break-up length of a liquid film was also developed by them. Four basic types of hydrodynamic instabilities were discussed: Tollmien-Schlichting, Kelvin-Helmholtz, Rayleigh-Taylor, and Bernard instabilities. These instabilities can act singly or together to alter the flow pattern.

Laminar-turbulent transition via the Tollmien-Schlichting mechanism occurs as a result of amplification,

by viscosity, of infinitesimal disturbances in the fluid. Transition occurs when the Reynolds number exceeds a certain critical value.

Kelvin-Helmholtz instability occurs in multi-phase flow systems where the fluid layers are in relative motion due to the interaction of the layers at the interface. The stability of stratified flows with a continuous velocity and density distribution depends on the parameter $J = -(g \, d\rho/dy)/\rho (du/dy)^2$ (Richardson number). The Reynolds number plays a role of only lesser importance. For fluids with discontinuous velocity and density profiles, the criterion becomes $J = g(\alpha_1 - \alpha_2)/\alpha_1\alpha_2(u_1 - u_2)^2$ where $\alpha_1 = \rho_1/(\rho_1 + \rho_2)$. The flow system becomes unstable when the disturbance wave number exceeds some value of the above parameters. In general, the interfacial shear force promotes instability whereas the inertia and surface tension forces promote stability. Kelvin-Helmholtz instability will occur no matter how small the velocity difference or shear between the two fluids.

The Rayleigh-Taylor type is an interfacial instability between two stratified fluids of different densities. Stability depends only on the relative orientation of the fluids.

The Bernard instability is associated with a single fluid within which an adverse density gradient is set up, as a result of a temperature gradient for example. The natural tendency for the fluid to redistribute itself is counteracted by its viscosity.

The effect of the Tollmien-Schlichting and Kelvin-Helmholtz instabilities are believed to be most important for fluids in with intermediate velocities.

The work of Benjamin(1) is in line with the findings of Gazley(14) and later, Charles(4,5) with respect to the delay in laminar-turbulent transition in one fluid caused by the presence of another fluid. Benjamin's work also appears to agree with the observations of Phillips(27). Benjamin considers theoretically the stability problem for flow past a flexible boundary, investigating the use of a flexible skin as a stabilizing device. He acknowledges three forms of instability. The first consists of unstable waves similar to those occurring in the presence of a rigid boundary. These waves tend to be stabilized when the wave velocity is less than the velocity of free surface waves on the boundary. Under these flow conditions the boundary has a "compliant response" to the waves. He found, however, that internal friction within the flexible medium was destabilizing. Benjamin found the second type of instability to be caused by a resonance effect; waves travel at nearly the same speed as free surface waves and are excited when the wave velocity falls below the free-stream velocity. The waves are amplified by the action of the flow supplying sufficient energy to overcome the internal viscous dissipation. The third form is similar to the Kelvin-Helmholtz instability previously discussed. It would appear

that the first form of instability discussed by Benjamin is responsible for the delay in transition to turbulence as observed by Gazley and Charles, whereas the second form is similar to the type observed by Phillips.

The foregoing review indicates that considerable knowledge has been acquired for single-phase flows. Laminar-turbulent transition generally occurs as a result of selective amplification of disturbances originating in the boundary layer or from fluctuations in the mean flow. Transition and turbulent flow in multi-phase systems are not well understood. The main effort has been directed toward the study of instability of the fluid-fluid interface, the system considered generally being air and water. Three more or less different types of instability, acting separately or combined, are important to the breakup of a fluid-fluid interface: first, the Tollmien-Schlichting instability of the mean flow; second, the instability occurring as a result of turbulent pressure and shear fluctuations; and third, the Kelvin-Helmholtz instability due to relative motion of the fluid layers. Theoretical instability analyses have not been completely successful. No single theory has incorporated all the relevant mechanisms of energy transfer. The particular type of instability effective at the interface is also believed to play an important part in establishing the turbulence pattern within the fluid phases.

III. SEMI-EMPIRICAL STABILITY CRITERION

A. Single Phase Flow

One of the earlier attempts to develop a simple general criterion for stability in fluid flow was made by Rouse(30). He proposed a dimensionless ratio $\chi = y^2 (du/dy)/\nu$, where y is the normal distance from the boundary, as a general index of laminar flow stability. The limit of stability is set by some constant critical magnitude where $\chi_{\max} = \chi_c$. The flow is said to be stable to all disturbances if $\chi_{\max}$ is below the value χ_c , and unstable to disturbances if the critical magnitude χ_c is exceeded. The value of $\chi_{\max}$ is directly proportional to the Reynolds number, but does not depend on the choice of the characteristic length term used in the calculations. Rouse obtained an average value for χ_c of about 500 for several flow configurations.

Ryan and Johnson(31) proposed a stability parameter for pipe flow, $Z_{\max} = \sqrt{4/27} N_{Re}$, evaluated at $r/r_w = \sqrt{1/3}$. This parameter is proportional to the critical Reynolds number for Newtonian fluids and is also applicable to non-Newtonian fluids of the power-law type. The critical parameter was found to be $Z_c = 808$ for Newtonian fluids in pipe flow, corresponding to a critical Reynolds number of 2100.

Hanks(16) proposed a generalized stability parameter which is independent of the flow system geometry and incorpor-

ated Ryan and Johnson's results as a special case. Hanks based his parameter on the premise that fluid motion will be unstable to certain types of disturbances if the magnitude of the acceleration forces acting on the fluid becomes a certain multiple of the magnitude of the viscous forces. The ratio of the forces becomes the stability parameter K . For rectilinear flows in steady state, this parameter can be expressed as:

$$K = \frac{\rho}{2} \frac{|\text{grad } (\vec{u} \cdot \vec{u})|}{|\vec{F} - \text{grad } P|} \quad (1)$$

The parameter K , like those of Rouse and Ryan and Johnson, is a function of position. It will vanish at the solid boundaries and at the position of maximum velocity. Somewhere in the flow field within the above limits, the parameter reaches a maximum, $K_{\max}$. For pipe flow this maximum will occur at $r/r_w = \sqrt{1/3}$. Only the maximum value, $K_{\max}$, is meaningful and is proportional to the bulk flow Reynolds number. Introducing the Poiseuille velocity profile expression into equation (1) and setting $r/r_w = \sqrt{1/3}$ yields for pipe flow:

$$K_{\max} = \sqrt{1/27} N_{\text{Re}} \quad (2)$$

If N_{Re} in equation (2) is set equal to the critical Reynolds number for pipe flow ($N_{\text{Re}_c} = 2100$), then $K_{\max}$ becomes the critical stability parameter

$$K_c = 404 \quad (3)$$

Note that K_c is equal to one-half of the value of Ryan and Johnson's critical stability parameter. Hanks shows that his critical parameter remains constant at 404 and applies to pipes, parallel plates, and concentric annuli.

For parallel plates, using the expression of plane Poiseuille flow with equation (1), one obtains

$$K = \left| \frac{3}{2} \frac{N_{Re}}{d_e} y \left[1 - \left(\frac{y}{h} \right)^2 \right] \right| \quad (4)$$

where y is the distance from the centre and h is half the plate separation. Substituting an equivalent diameter equal to four times the hydraulic radius, $d_e = 4h$, equation (4) becomes

$$K = \left| \frac{3}{8} N_{Re} \frac{y}{h} \left[1 - \left(\frac{y}{h} \right)^2 \right] \right| \quad (5)$$

For $K = K_{max}$, at $dK/d(y/h) = 0$, $y/h = \sqrt{1/3}$. Therefore, from equations (5) and (3) a critical Reynolds number N_{Re_c} equal to 2800 can be calculated for parallel plates. This agrees with the experimental findings of Davies and White(9) who also found N_{Re_c} (parallel plates) equal to 2800.

B. Two-Phase Flow Between Parallel Plates

The laminar velocity profile for two immiscible, incompressible, Newtonian fluids flowing co-currently between

two parallel plates (Figure 2a) can be given by the following expressions:

$$u_1 = \frac{g_c}{2\mu_1} \left(\frac{\Delta P}{\Delta Z} \right) \left[y^2 - Ay - b_1(b_1 + A) \right] \quad (6a)$$

for $-b_1 \leq y \leq 0$

$$u_2 = \frac{g_c}{2\mu_2} \left(\frac{\Delta P}{\Delta Z} \right) \left[y^2 - Ay - b_2(b_2 - A) \right] \quad (6b)$$

for $0 \leq y \leq b_2$

where

$$A = \frac{\mu_1 b_2^2 - \mu_2 b_1^2}{\mu_1 b_2 - \mu_2 b_1}$$

The flow is assumed to be horizontal, fully developed and the fluid interface is smooth. Flow is in the z-direction. The y-coordinate is chosen perpendicular to the flow and is zero at the interface. The liquid depths are b_1 and b_2 respectively with liquid 2 flowing above liquid 1. The volumetric flow rate for each of the two phases is then obtained by integrating the velocity distribution with the following results:

$$Q_1 = \frac{-g_c}{2\mu_1} \left(\frac{\Delta P}{\Delta Z} \right) \left(\frac{2}{3} b_1^3 + \frac{b_1^2 A}{2} \right) \quad (7a)$$

$$Q_2 = \frac{-g_c}{2\mu_2} \left(\frac{\Delta P}{\Delta Z} \right) \left(\frac{2}{3} b_2^3 - \frac{b_2^2 A}{2} \right) \quad (7b)$$

Note that if $\mu_1 = \mu_2$ and $b_1 = b_2$ equations (6a,b) and (7a,b)

simplify to the case of single phase laminar flow between parallel plates.

The stability parameter is directly related to the velocity distribution in a single phase flow equation. To adapt the parameter to a two-phase system, calculations must be carried out on each phase separately. In so doing, however, it is assumed that the one phase has no stabilizing or destabilizing influence on the other and that the two phases are connected only by the established overall velocity distribution.

The stability calculations for this two-phase system were based on a hypothetical plate separation. A plate separation can be calculated such that the actual velocity profile in each phase corresponds to part of a hypothetical, single phase velocity profile. An example of the hypothetical separation and velocity profile is illustrated in Figure 2b. Using this method, it is possible to perform stability calculations of the two-phase system by treating each phase as a separate, single phase.

The hypothetical plate separation, $2h$, can be determined from the expressions for velocity. From Figure 2a it can be seen that

$$u_1 = 0 \quad \text{at } y = -b_1 \text{ and } y = -b_1 + 2h_1$$

$$u_2 = 0 \quad \text{at } y = b_2 \text{ and } y = b_2 - 2h_2$$

From equation (6a), at $y = -b_1 + 2h_1$

$$y^2 - Ay - b_1(b_1 + A) = 0$$

Therefore

$$h_1 = b_1 + \frac{A}{2} \quad (8a)$$

From equation (6b), at $y = b_2 - 2h_2$

$$y^2 - Ay - b_2(b_2 - A) = 0$$

Therefore,

$$h_2 = b_2 - \frac{A}{2} \quad (8b)$$

The two-phase flow system can now be represented by two single-phase flow systems. Fluids 1 and 2 flow between parallel plates separated a distance of $2h_1$ and $2h_2$ respectively. The actual plate separation of the two-phase system is $H = h_1 + h_2$.

If equations (6a,b) are substituted into equation (1), expressions for the local K parameter for each phase are obtained.

$$K_1 = \left| \frac{g_c}{4} \frac{\rho_1}{\mu_1} \left(\frac{\Delta P}{\Delta Z} \right)_1 \left[y^2 - Ay - b_1(b_1 + A) \right] (2y - A) \right| \quad (9a)$$

$$\text{for } -h_1 \leq y \leq h_1$$

$$K_2 = \left| \frac{g_c}{4} \frac{\rho_2}{\mu_2} \left(\frac{\Delta P}{\Delta Z} \right)_2 \left[y^2 - Ay - b_2(b_2 - A) \right] (2y - A) \right| \quad (9b)$$

$$\text{for } -h_2 \leq y \leq h_2$$

Since only the maximum value of K_i , $i = 1, 2$, is meaningful as a stability criterion, the position where this maximum occurs must be determined for each hypothetical plate separation.

With the position variable y equal to zero at the interface, the point where $K = K_{\max}$ becomes a function of the interface position as well as the hypothetical plate separation.

$$\bar{y}_1 = h_1(1 - \sqrt{1/3}) - b_1 \quad (10a)$$

$$\bar{y}_2 = b_2 - h_2(1 - \sqrt{1/3}) \quad (10b)$$

Setting $y_1 = \bar{y}_1$, $y_2 = \bar{y}_2$, and assuming unit pressure gradient $K_{1,\max}$ and $K_{2,\max}$ can be calculated from equations (9a,b). The stability parameter indicates that for laminar-turbulent transition flow $K_{i,\max}$ is constant and equal to 404. The transition pressure gradient can, therefore, be calculated for each phase.

$$\left(\frac{\Delta P}{\Delta Z}\right)_{1,\text{transition}} = \frac{404}{K_{1,\max}} \quad (11a)$$

$$\left(\frac{\Delta P}{\Delta Z}\right)_{2,\text{transition}} = \frac{404}{K_{2,\max}} \quad (11b)$$

Equations (11) represent the pressure gradient for the two-phase system at which turbulence will begin to develop in each phase. Critical, superficial Reynolds numbers can now be calculated for any interface position. The critical pressure gradient from equations (11a,b) can be used in equations (7a,b) to calculate the critical volumetric flow rate, from which, in turn the superficial critical Reynolds number is obtained.

$$N_{Re_c} = \frac{2Q_i}{v_i}, \quad i = 1, 2 \quad (12)$$

C. Two-Phase Flow in a Rectangular Conduit of Large Aspect Ratio

An approach similar to that described above can be used to calculate a point of transition for co-current flow of two immiscible fluids in a rectangular conduit of large aspect ratio.

Charles(4) has developed expressions for the velocity distribution and volumetric flow rate of two immiscible fluids in a rectangular conduit (Figure 2c).

$$u_j = \frac{16a^2}{\pi^3} \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i+1)^3} \left[A_j \sinh(ny) + B_j \cosh(ny) \right] \cos(nx) \\ + \frac{k_j}{2} (x^2 - a^2) \quad (13)$$

for $j = 1, 2$, where

$$k_j = \frac{g_c}{\mu_j} \left(\frac{\Delta P}{\Delta Z} \right)$$

$$m = \frac{\mu_2}{\mu_1}$$

$$n = \frac{(2i+1)\pi}{2a}$$

$$A_1 = mA_2$$

$$A_2 = \frac{k_2 \cosh nb_1 - k_1 \cosh nb_2 + (k_1 - k_2) \cosh nb_1 \cosh nb_2}{(\cosh nb_1 \sinh nb_2 + m \sinh nb_1 \cosh nb_2)}$$

$$B_1 = \frac{k_1 \sinh nb_2 + mk_2 \sinh nb_1 + m(k_1 - k_2) \sinh nb_1 \cosh nb_2}{(\cosh nb_1 \sinh nb_2 + m \sinh nb_1 \cosh nb_2)}$$

$$B_2 = \frac{mk_2 \sinh nb_1 + k_1 \sinh nb_2 - (k_1 - k_2) \sinh nb_2 \cosh nb_1}{(\cosh nb_1 \sinh nb_2 + m \sinh nb_1 \cosh nb_2)}$$

$$Q_1 = \frac{128a^4}{\pi^5} \sum_{i=0}^{\infty} \frac{1}{(2i+1)^5} \left[A_1 (1 - \cosh nb_1) + B_1 \sinh nb_1 \right] - \frac{2}{3} k_1 a^3 b_1 \quad (14a)$$

$$Q_2 = \frac{128a^4}{\pi^5} \sum_{i=0}^{\infty} \frac{1}{(2i+1)^5} \left[A_2 (\cosh nb_2 - 1) + B_2 \sinh nb_2 \right] - \frac{2}{3} k_2 a^3 b_2 \quad (14b)$$

The superficial Reynolds number is given by

$$N_{Re} = \frac{Q_i}{\dot{v}_i \left(a + \frac{H}{2} \right)} ; \quad i = 1, 2 \quad (15)$$

For a conduit of large aspect ratio it can be assumed that the velocity distribution at the center ($x = 0$) is two-dimensional. Therefore, the parameter, K , is maximum at the

horizontal center and at the same position ($y = \bar{y}$) as for parallel plates. Critical Reynolds numbers are again calculated for both phases in the manner indicated for parallel plates. Calculations, however, are more difficult due to the complicated nature of the velocity distribution and flow rate expressions.

D. Discussion of Calculated Results

Calculations were performed on the University IBM 7040 computer. The basic aim was to establish loci of the flow rates resulting in laminar-turbulent transition in a co-current two-phase, liquid-liquid system and to compare these results with experimental findings. Computations were made for two liquids, designated as 1 and 2, flowing between parallel plates and in a rectangular conduit. Both the parallel plates and the rectangular conduit were assigned a plate separation of one inch with the latter having an aspect ratio of eight to one. The two liquids were assumed to flow co-currently with liquid 2 above liquid 1. The interface was considered horizontal, smooth and sharply defined. Physical properties of water and a mineral oil "Mentor 29", at 80°F were used for the calculations (Table 1).

In the development of the previous section it was assumed that for the theoretical laminar-turbulent transition calculations a rectangular conduit of large aspect ratio can

be treated similar to parallel plates. The validity of this assumption was verified in a recent theoretical study by Hanks and Ruo(17). These authors calculated the lower critical Reynolds number as a function of the duct aspect ratio for steady, isothermal flow of Newtonian fluids in straight ducts with constant, rectangular cross-section. The same generalized parameter for the calculation of the lower critical Reynolds number as used in this investigation for a two-phase flow system was used by Hanks and Ruo for a single-phase system. The results of their computations are presented in Figure 3 and 4. Figure 3 represents the theoretical values of the critical Reynolds number as a function of the aspect ratio. Experimental data points obtained from the literature are also shown and verify the theoretical calculations. Figure 4 is a plot of the spacial coordinate $\bar{y}/h$ as a function of the aspect ratio. The coordinate gives the theoretical location of turbulence inception which corresponds to the position where the parameter K is maximum. Figure 4 shows that for a duct of aspect ratio eight to one, as used in the calculations, the location of turbulence inception is indeed the same as for parallel plates, namely $\bar{y}/h = \sqrt{1/3}$. Hanks and Ruo further indicate that the location of turbulence will, with one exception, always be at $x/a = 0$. The exception is a conduit of aspect ratio one; the turbulence inception point will be at $y/h = x/a = 0.637$.

Figure 5 shows the stability parameter for water in two-phase flow with oil in a rectangular conduit as a function of the horizontal distance x/a . The point of turbulence inception was at $\bar{y}/h = \sqrt{1/3}$, which corresponds to 0.1376 inches from the lower conduit plate. The value of $K_{\max}$ remains constant at 404 for the central section of the duct and starts to decrease to zero as the lateral wall is approached. The assumptions that K for a rectangular duct is maximum at the horizontal center ($x/a = 0$) and at the same position ($y/h = \bar{y}/h$) as for parallel plates can, therefore, be considered as verified.

Loci of the critical superficial Reynolds numbers for both water and oil were calculated as a function of interface position, following the method outlined in the foregoing sections. The results are tabulated in Table 2 and are illustrated in Figure 6 which is a plot of the calculated oil Reynolds number versus the calculated water Reynolds number. Values are given for both parallel plates and the rectangular conduit. Interface position is used as a parameter. Also plotted on Figure 6 are some of the experimental results for turbulence transition in the water phase. Detailed reference to these values will be made later in the section titled "Intermittent Flow". However, it can be pointed out here that the agreement between the calculated and experimental transition points for the water phase is good for interface positions greater than 0.4. As expected, agreement is better for the

rectangular conduit than for parallel plates. Note that the calculated transition Reynolds numbers correspond to the lower critical Reynolds numbers, whereas the experimental values correspond to γ equal 0.5. Figure 6 can also be compared with the experimental findings of Charles(4) as given in Figure 1.

Agreement exists for transition in the water phase for interface positions greater than 0.4 and in the oil phase for interface positions less than 0.1. The theoretical, single phase lower critical Reynolds number is 2300, as determined experimentally by Charles.

The above stability parameter, thus, appears to be successful in predicting laminar-turbulent transition in a two-phase system only if the phase under consideration occupies a large fraction of the conduit flow area. Figure 7 presents the velocity distribution at the central cross-section of the rectangular conduit for various oil-water interface position. From Figures 6 and 7 it appears that the stability parameter is valid if the maximum velocity occurs in the phase for which stability calculations are being performed. Under such conditions, the main fluid layer, containing the point of maximum velocity, is not influenced significantly by the presence of the other layer. The other fluid layer only reduces the effective dimensions of the conduit. When the transition to turbulence of the phase not containing the maximum point of velocity is considered interfacial forces play an important role.

As the stability parameter is based on a ratio of the acceleration forces to the viscous forces within a flow system, the criterion can predict transition only for systems where these forces represent the major agents of instability. Energy transfer from one phase to the other via interfacial shear stress or resonance pressure fluctuations is not considered in the above parameter. In order to account for the additional variables introduced with the multi-phase system, the critical magnitude of the K parameter will have to become dependent upon the gravitational forces, interfacial tension, and shear as well as the density and viscosity ratios. This brief study has pointed out the importance of the energy transfer occurring at the interface, on the laminar-turbulent transition in two-phase flow systems.

IV. EQUIPMENT

The basic flow equipment used in this investigation was previously designed and constructed by Charles as part of his study of stratified liquid-liquid flow at the University of Alberta. A detailed description of each component part is given in his Ph.D. thesis(4). The overall flow system is illustrated in Figure 8.

A. Fluids

The liquids chosen for this experimental work were de-ionized tap water and a mineral oil, Mentor 29, supplied by Imperial Oil Ltd. This oil was one of the binary oil mixture used by Charles. The fluids had a density ratio of 0.825 and a viscosity ratio of 5.58 at 80°F. These properties resulted in ready separation of the liquids by gravity and permitted the use of wide differences in individual flow rates. The interfacial tension was 40.6 dynes/cm at 74°F. Table 1 gives the liquid densities and viscosities as a function of temperature.

B. Rectangular Conduit

The rectangular conduit was constructed of "Lucite", and had an overall length of 37.5 feet with a nominal inside width and height of eight and one inches respectively.

Its average aspect ratio was 7.95:1 as determined by Charles from pressure drop measurements. The entry section was constructed of sheet metal and had an overall length of 33 inches. The two liquids entered the section at the top and bottom of a horizontal circular cylinder which gradually changed its cross-section to the rectangular cross-section of the main conduit. Internal, horizontal baffles channeled the incoming liquids in the proper flow direction minimizing mixing at the interface. The sheet metal discharge section was 2 feet long and gradually changed from a rectangular to a circular cross-section. Pressure taps were located along the center line of the lower conduit plate 12.5 and 32.5 feet from the entry section.

The conduit had also an 18 inch removable test section situated 24 feet from the entrance. This section was previously used for photographic purposes(4), for velocity profile measurements(35) and for photometric measurements(37). A horizontally and vertically traversing pitot tube mechanism(35) was used as a pointer gauge to measure the liquid-liquid interface position. The vertical movement of the pointer gauge was measured by a micrometer screw. A two-inch removable plug used in the photometric work was located in the center of the upper plate of the test section, 5 inches from its upstream end.

C. Hot-Film Probe Traversing Mechanism

A diagram of the hot-film probe installation is shown in Figure 9. The hot-film probe manufactured by Lintronic Laboratories consisted of a 4-mm Pyrex glass rod 10 cm long with two 32-gage platinum lead wires embedded in its core. A 30 degree cone formed one end of the glass rod, and a micro-centimeter thin platinum film, with a total resistance of 20 ohms was fused to the tip. The film was connected at its opposite edges to the two platinum leads. Thus, electric current passing through the leads would heat the thin platinum film which forms the sensing element. The wire leads from the probe were drawn through a 0.25 inch O.D. L-shaped stainless steel tube and the glass probe was mounted into the short leg of the tube. The legs of the L-shaped tube were one and ten inches long.

An oblong opening 0.75 inches wide 3.70 inches long, with semi-circular ends, was cut into the upper plate of the removable test section. The oblong hole was positioned at 45 degrees to the flow direction with the center of the semi-circular end at the center of the upper plate and at the upstream end of the test section. The 45 degree angle was required because of the obstruction of the 2-inch photometric plug mentioned in Section B "Rectangular Conduit". A "Lucite" plug, sealed with an O-ring, and equipped with the probe traversing mechanism, was placed into the above opening and

fastened to the upper plate with two Allen screws. The stainless steel tube, also sealed with an O-ring passed through the plug, such that the hot-film probe proper was positioned in the centre of the conduit pointing upstream.

The traversing mechanism (Figure 9) consisted of a vertically mounted micrometer screw and a dovetailed slide block onto which the stainless steel stem of the probe was fastened with two set screws. The micrometer screw was fixed at the top and the spindle controlled the vertical movement of the slide block and the probe.

D. Auxiliary Flow Equipment

A 52-inch diameter, 75-inch high cylindrical tank functioned as a combination separator and storage tank for the two experimental liquids. The separator was constructed from galvanized iron sheet and had a total capacity of approximately 650 U.S. gallons. Concentric baffles as illustrated in Figure 8 aided in the separation of the liquids. The inner baffles were 27 inches in diameter and 25 inches high; the outer one was 42 inches in diameter and 52 inches high.

Positive displacement gear type pumps for the oil and water took suction at the center of the inner baffles at the top and bottom of the separator. The oil pump had a capacity of 36 U.S. gallons per minute at 1750 R.P.M. and was model K30 manufactured by GD Roper Corporation.

The water pump had a capacity of 30 U.S. gallons per minute at 1750 R.P.M. and was model FL manufactured by Jabsco Pumps Company.

Unwanted portions of the total flow from each pump could be recycled directly to the separator or passed into individual bypass tanks for oil and water. The bypass tanks were identical for both phases; being 35 inches high and 22 inches in diameter with capacities of 50 U.S. gallons. Bypass tanks were required because high bypass flows returned directly to the separator impeded the liquid-liquid separation. The required portion of each phase was metered with three parallel rotameters. For each phase the two large ones were Brooks Rotameters and the smaller one was a Fischer-Porter Rotameter. The liquids from the rotameters passed through a double-pipe heat exchanger for temperature equalization. The oil phase then entered the rectangular conduit directly whereas the water phase entered the conduit after passing through a Y-type, 400 mesh filtering screen. The filtering screen was necessary to remove any solid particles which could damage the hot-film sensing element.

The oil and water flowed through the conduit and discharged tangentially into the separator. Gravitational separation occurred in the annulus between the outer concentric baffle and the tank wall.

A 5-inch bed of coarse sodium chloride, supported on a perforated tray in the oil phase between the inner and outer baffle, removed water droplets entrained in the oil.

The oil-water interface in the separator could be observed in a sight glass and was maintained at the vertical center of the rectangular conduit. Make-up water passed through a model GM 18 Lindsay Water Softener and entered the separator through three 0.5-inch openings spaced equally along the tank circumference.

Four thermometers were used to monitor the temperatures of the liquids. Two mercury thermometers were placed in openings in the separator wall, one in each phase. Two dial thermometers, again one in each phase, were present in the conduit entry section.

A U-tube, 10-mm glass manometer was used to measure pressure drop over the 20 foot conduit section. The manometer fluid was a 2.5:1 mixture of benzene and carbon tetrachloride. The solubility of these liquids in water is 0.08 parts per 100 parts at 20°C. The change in density of the water legs in the manometer, due to dissolved organic liquid was considered negligible. The density of the manometer fluid as determined by a Christian Beckers Chain Gravitometer was 1.0994 gm/cc at 79°F. The liquid level difference in the manometer tubes was measured with a cathetometer to an accuracy of ± 0.005 cm.

E. Electronic Equipment

1. The Linear Constant-Temperature Hot-Film Anemometer

The hot-film anemometer is an instrument used to study the flow characteristics of both gases and liquids. The basic operating principles are similar to the well-known hot-wire anemometer. The main difference lies in the sensing element. The hot film is an extremely thin platinum film (50 to 100 Angstroms) fused to the tip of a rugged glass probe, whereas the hot wire is a fragile self-supporting element.

The hot-wire anemometer, was originally developed for air flow measurements, but can be used directly in liquid flow measurements if ideal conditions are maintained. The liquid and piping materials should be electrically non-conducting (10^6 ohms/cubic centimeter) and the liquid must be free of gas bubbles, microscopic dirt and other non-homogeneities. The wire is very sensitive to microscopic surface contamination. The above requirements could not be met in the experimental system of this investigation. The requirements of the hot-film anemometer are less stringent. The larger, more uniform heat transfer surface of the hot film is less sensitive to minute quantities of impurities collecting on the tip. This type of instrument was, therefore, employed to study the turbulence characteristics in laminar-turbulent

transition flow of water.

The instrument used was a Linear Constant-Temperature Hot-Film Anemometer Model 40W supplied by Lintronic Laboratories. Details on the design and operations of this instrument are given in references (22) and (23). The anemometer had both A.C. and D.C. outputs; the electrical signal being proportional to the fluctuating and mean flow velocity components respectively. The D.C. output voltage could be damped to any desired level by connecting an external capacitor. The instrument consisted of two separate units, a regulated power supply unit and the temperature control and linearizing unit. The power source to the combined system was maintained at a constant 118 volts by a Sola Constant Voltage Transformer. A cone-shaped hot-film probe was chosen because the flow pattern around it minimized the collection of microscopic dirt and gas bubbles which affect the stability of the signal.

a. Basic Theory

In measuring the instantaneous velocity of a fluid the hot-film technique utilizes the forced convective heat transfer from the electrically heated metal film to the fluid medium in which it is immersed. The rate of heat loss from the film is related to the temperature difference between the hot film and the fluid. The temperature of the film, in turn, is linearly related to its electrical resistance. Thus, the hot film temperature changes, caused by changes in fluid

velocity, are converted into electrically measurable resistance changes.

Heat transfer from the film occurs in two parts: by forced convection to the moving fluid and by conduction through the film support material. The conductive heat transfer can be further separated into a steady state heat loss and a dynamic heat loss or gain of the film. Thus, the heat generated in the film, $I^2 R$, where I is the film current and R the film resistance, is equal to the convective plus the conductive heat loss from the film.

There are generally two techniques used in anemometer operations. If the heating current I is kept constant, the resistance R becomes a function of the fluid velocity. However, if R is kept constant by means of a servo control system, I becomes a function of the velocity. This latter technique, known as the constant temperature method, was applied in the instrument used in this investigation.

For constant temperature operation, the equation

$$I^2 = A\sqrt{u} + B \quad (16)$$

represents the static signal response of the hot-film sensing element. The constants A and B are empirical. Equation (16) above is nonlinear. In the linear constant temperature anemometer the signal I is reprocessed through a nonlinear electronic device to obtain the voltage as a linear function

of velocity,

$$E = C u + D \quad (17)$$

where C and D are calibration constants.

Constant temperature operation of a hot film improves its frequency response. If the hot-film temperature is kept perfectly constant, the dynamic conduction heat loss or gain of the film is zero and the frequency response will be uniform. However, a temperature control system which can maintain an absolutely constant film temperature is not possible as a finite error or deviation must occur to bring about a controlling action. The dynamic response of the sensing element must be given consideration in the design of the control circuits. A block diagram of the control system and basic circuitry of the anemometer are illustrated in Figure 10. The hot film is used as one arm of a Wheatstone bridge with fixed resistors in the other arms. The negative feedback control loop keeps the bridge balanced resulting in a constant film temperature.

2. Other Electrical Measuring Devices

A two-channel Beckman Type RS Dynograph direct writing recorder was utilized to record the anemometer output signal. The Dynograph was used in conjunction with AC/DC Input Couplers 9806A which are designed to provide selective control over combined AC and DC voltage signals. The combined

signal or the AC signal alone could be admitted for measurement. The coupler could also provide selectable attenuation of high-frequency or low-frequency parts of the input signal.

In recording the fluctuating voltage signal from the anemometer, the DC signal and fluctuation frequencies of less than 0.16 cps were damped out. The upper frequency cut off was 150 cps. The signal fed to the Dynograph was traced on heat sensitive paper; each channel having a linear recording range of 5 cm. The incoming signal was attenuated to stay within the above deflection range. Chart speeds of 1, 5, 25 and 125 mm per second were possible.

A Flow Corporation model 12A1 Random Signal Voltmeter was utilized for turbulence intensity measurements. The voltmeter had a time constant of 16 seconds and a flat frequency response from 2 cps to 250 kcps. The lower limit of the frequency range was found adequate only for water Reynolds numbers greater than 3000.

A Lintronic RMS Voltage Analyser, which had a relative gain of one for frequencies between 0.3 and 20,000 cps, was also available. This instrument, however, failed to function properly and its use was discontinued.

During preliminary studies, a dual beam Tektronix Type 502 Oscilloscope was used to observe voltage fluctuations and to study the signal noise level. To minimize the noise problem all leads were shielded and all instruments were grounded at a common point.

V. PROCEDURE

A. Preliminary

Preliminary experiments performed on a small, bench scale conduit indicated that the hot-film anemometer was a suitable instrument for detecting the laminar-turbulent transition region in water. An attempt was made to obtain a direct meter reading of the intermittency factor in the transitional flow regime. An electronic circuit similar to that described by Bradsbury(3) was built to analyse the anemometer output signal. Difficulties due to the low frequency turbulence fluctuations were, however, encountered and the intermittency meter could not be made operational. It was, therefore, decided to use the direct writing Dynograph recorder to record the anemometer output signal and to analyze the traces manually. The liquid separator and conduit were thoroughly cleaned and repaired. The interior of the sheet metal entry and exit sections were coated with a zinc-base paint. The separator was then filled with water. The water was circulated throughout the system to rinse the piping and was then drained. Fresh water and oil were added to the separator tank. The oil-water interface was position at the midpoint of the conduit outlet and the oil level was set five inches above the oil outlet. These levels minimized the oil and water entrainment near the interface of the returning

liquids and prevented the oil pump from drawing air.

The water rotameter calibrations obtained by Charles(4) were checked and found to agree within one percent. The oil rotameters were completely recalibrated as the oil had different properties from that used by Charles. The calibration procedure outlined by him was also used in this investigation.

After the final adjustment of the water and oil levels in the separator tank had been made, the water was recirculated continuously through the pump, bypass tank and the conduit for approximately four hours. The accompanying gradual temperature increase of the water, combined with the extreme agitation, caused dissolved air in the water to come out of solution. The presence of air in the water phase was undesirable as it resulted in the formation of small bubbles on the hot-film sensing element. Further, a corrosion problem existed in the separator tank which could be aggravated by the presence of oxygen in the water. As a result fine rust particles became suspended in the water and settled on the conduit wall and on the probe tip. Temperature control of the water was not possible by simply purging a fraction of the warm water from the storage tank and adding cold tap water as make up.

B. Experimental

The hot-film probe was disconnected from the anemometer unit and removed from the conduit before each experimental run. The hot-film element was first cleaned with carbon tetrachloride to remove any oil film, then dipped in concentrated hydrochloric acid and rinsed with water to remove other contaminants collected in the water. The probe was again inserted, fastened into the conduit and connected to the control unit. The desired vertical level of the probe was then set using the micrometer screw. The Y-filtering screen at the water inlet was removed, cleaned and replaced.

The liquids were allowed to fill the conduit under gravity. Water was first introduced into the conduit by opening the inlet valve and the vent on the top of the entry section. When the water completely covered the bottom of the conduit and the hot-film probe, the oil inlet valve was opened. Care was taken to prevent the oil from contacting either the probe or the bottom of the conduit. When the conduit was full, the vent was closed, the conduit exit valve was opened and the water and oil pumps were started. The water and oil flow rates were then set to the desired values by adjusting the respective bypass and rotameter valves.

The hot-film anemometer and Dynograph recorder were next placed in operation. Flow and instrument operation were considered steady when the anemometer output became

reasonably stable. The AC and DC anemometer output signals were then recorded for 800 seconds for each set of flow conditions. During this time the water depth in the conduit was measured using the pointer gauge mechanism described earlier. The pressure gradient over the 20-foot section, and the conduit inlet oil and water temperatures were also recorded. At low flow rates the pressure measurements are not believed to be accurate. Under these conditions, the surface tension effects, due to impurities in the manometer tube, may have resulted in some error.

During each experimental run the oil flow rate was maintained constant. The water rate was increased incrementally over the entire laminar-turbulent transition regime. The oil flow rate was then altered and the above procedure was repeated.

In the shut-down procedure, all the electronic equipment was turned off first. The oil pump was then shut off; the oil inlet valve was closed and the water was allowed to wash the oil layer out of the conduit. The water pump was then turned off; the conduit exit valve and the water inlet valve were closed. The conduit was then drained slightly to relieve the pressure and to permit removal of the hot-film probe.

VI. DISCUSSION OF EXPERIMENTAL RESULTS

The initial problem of this investigation was to develop a method of defining a flow rate or Reynolds number which would characterize the transition from a laminar to a turbulent flow condition in single- and two-phase systems. One method of detecting laminar-turbulent transition is by observing the velocity distribution in the flow field. Velocity fluctuations characteristic of turbulent flow as mentioned earlier, can be detected and measured from an Eulerian point of view (observer stationary) using a hot-wire or a hot-film anemometer.

A. Turbulent Intensity Measurements

It was believed that the rate of change of the relative intensity of turbulence with respect to increasing liquid velocity, would be different in laminar and transition flow than in turbulent flow. A parameter proportional to the relative intensity of turbulence was, therefore, obtained by measuring the root-mean-square (RMS) value of the fluctuating output signal from the hot-film anemometer. The RMS voltage reading was assumed to be proportional to the velocity fluctuation intensity. As an absolute value of the relative intensity of turbulence was not required, no anemometer calibration was necessary. It was, however, essential that the sensitivity

of the hot-film probe remain constant for the duration of an experimental run in order that successive RMS voltage readings could be compared.

A series of trial runs was made to evaluate the suitability of the turbulence intensity measurements for the characterization of the transition process. The hot-film probe was positioned in the center of the rectangular conduit and the water flow rate was increased in small increments from a Reynolds number of 500 to about 15,000. For these trial runs there was no oil in the conduit. The RMS voltage of the anemometer signal was recorded for each water flow rate. The logarithm of the RMS voltage was then plotted versus the corresponding average water velocity as shown in Figure 11, Table 3. This Figure illustrates that the experimental points can be represented by two intersecting straight lines. The average velocity at the point of intersection can be interpreted to represent the upper limit of the transition region. For single phase water flow in the rectangular duct this corresponds to a transition Reynolds number of 3100. This value is in good agreement with the upper critical Reynolds number for single phase water flow as determined by intermittency measurements discussed in the section titled "Intermittent Flow".

In order to determine the position in the stream cross-section where turbulence first occurs, measurements

of the intensity of turbulence were taken at various horizontal levels along the central, vertical cross-section. Turbulence inception is considered to occur at the position where the highest turbulence intensity is detected. The water flow rate was kept constant for the entire traverse. The hot-film probe tip could be brought only within 0.180 inches of the upper and lower conduit wall due to the large diameter of the steel tube used to support the hot-film probe.

The RMS voltage, which is proportional to turbulence intensity, was lowest at the center and maximum near the walls of the conduit. Gibson(15) and Leite and Kuethe(19) observed maximum velocity fluctuations in pipe flow at r/r_w approximately equal to 0.6. Thus, the distance from the wall where maximum fluctuation intensity occurred was $0.4r_w$. Thus, by analogy, in a rectangular conduit the point of least stability or maximum velocity fluctuation should occur at $0.4h$ or, for the conduit of this investigation, 0.2 inches from the conduit wall. Further, the stability parameter calculations discussed in Section II give the position of maximum instability at 0.212 inches from the wall. This position corresponds roughly to the measurements taken nearest the solid boundary of the conduit. Presumably, had it been possible to take turbulence intensity measurements closer to the conduit wall, fluctuating velocity components would again decrease and finally approach zero near the boundary. The

maximum velocity fluctuations can reasonably be assumed to occur at approximately 0.2 inches from the wall. All further basic experimental measurements were taken with the hot-film probe near the lower plate of the rectangular conduit because:

1. this position could be reproduced accurately;
2. maximum velocity fluctuations occurred at this position;
3. for two-phase flow, the liquid-liquid interface could be brought as low as possible without changing the probe position and still maintaining the sensing element entirely within the water phase.

The above measurements of turbulence intensity using the root-mean-square volt meter, were discontinued after a series of trial runs. Reproducible measurements of the turbulence intensity as indicated by the RMS voltage were difficult to obtain. In the transition flow regime, the voltage readings fluctuated widely as a result of the intermittent flow. The turbulence intensity increased rapidly during the duration of a turbulent spot and decreased during the laminar flow period. Neither the laminar nor the turbulent fractions of the flow field were generally maintained long enough for the RMS-voltmeter to come to steady state. An average reading, between the upper and lower limits of fluctuation was not representative because the length of the turbulent spots changed with changing flow rates. The local physical and electrical properties of the water varied with

time and the liquid temperature increased slowly. Thus, although the volumetric flow rates could be maintained constant, the Reynolds number changed noticeably with time due to changes in the physical properties of the fluids. Heat exchange equipment did not function satisfactorily. In addition, the hot-film probe gradually lost sensitivity as the result of fine rust and dirt particles settling on the platinum tip. The anemometer voltage output also drifted due to the decreasing temperature difference between the hot-film and the liquid. Further, the flow system appeared to be subject to the formation of resonant frequencies set up by the liquid feed pumps. These frequencies appeared to change with different control valve settings resulting in different levels of apparent background turbulence intensity. As a result, the RMS voltage readings were erratic and difficult to reproduce. In addition, over the range of Reynolds numbers considered, the turbulence signal encountered was of very low frequency (between 0.5 and 5 cycles per second). The range of linear frequency response of the available RMS voltmeter did not fully cover these low frequencies resulting in further errors.

B. General Characteristics of Fluctuating Velocity Components

In view of the difficulties encountered in determining the turbulence intensity measurements, it was decided to use a different method to study the transition from laminar

to turbulent flow in liquids. As no satisfactory means to calibrate the anemometer was available and as the anemometer output signal stability was a problem, a fully quantitative study of the liquid fluctuation characteristics was not possible. However, the Dynograph recorder described in Section IV C 2 was found to be very suitable in recording the hot-film anemometer output signal. Both the D.C. and A.C. signals from the anemometer could be recorded simultaneously. The D.C. voltage output was proportional to the time average liquid velocity past the hot-film probe and the A.C. voltage output was proportional to the fluctuating velocity components past the probe. The proportionality constant in each case was unknown. Comparisons of relative amplitude of the fluctuating velocity components could be made only for records taken within a relatively short time period of less than ten minutes. During such a short period the anemometer output drift and change in probe sensitivity, as well as changes in the physical and electrical properties of the liquid, could be assumed negligible. No attempt was made in this study to measure the directional nature of the velocity fluctuations.

A series of preliminary runs, using the Dynograph recorder in conjunction with the hot-film anemometer, was performed to establish some of the general characteristics of single and two-phase laminar, transitional, and turbulent liquid flows. No experimental information for similar liquid

systems was found in the literature.

The transition from laminar to turbulent flow was found to occur in three more or less distinct stages or phases. The first phase was typified by the formation and growth of regular sinusoidal velocity fluctuations. In the second phase the sinusoidal fluctuations developed into sharply defined spots of irregular or turbulent motion. In the third phase the turbulent spots expanded and grew until the entire flow regime was in a fully random turbulent flow. Figure 12 shows typical records of the velocity fluctuations in these three regimes. These records were obtained for water Reynolds numbers of 1000, 2500, 3000, and 4000 respectively. The oil flow rate was zero in each case, and the hot-film probe was positioned midway between the side walls and 0.180 inches from the lower conduit plate. Figure 12 gives some indication of the fluctuating velocity component frequencies. The time scale on these tracings is 5 mm/sec. However, accurate estimates of the fluctuation amplitudes cannot be made as the anemometer was not calibrated and the traces were taken at different times.

The overall process of transition to turbulence in the water phase in the presence of the oil phase was similar to that described above. However, a number of interesting and important differences did exist as will be discussed in more detail in Section VI B 2 and 3.

1. Sinusoidal Velocity Fluctuations

A further illustration of the sinusoidally fluctuating velocity during the first stage of the transition process as encountered in the single phase flow of water in the rectangular conduit, is presented in Figure 13 a, b, and c. The recordings were again made with the hot-film probe at the center of the conduit and 0.180 inches from the lower plate. Figure 13 a and b correspond to a water Reynolds number of 260 and 2540 respectively. Both records were taken within a short time period (less than 10 minutes) and all conditions, except the water flow rate, were maintained constant. The relative amplitude of the output signal is, therefore, an indication of the relative amplitude of the velocity fluctuations at the two different flow rates. At water Reynolds number 2540 the fluctuating velocity amplitude is approximately 1.5 times that at Reynolds number 260. The sinusoidal oscillations appear to be present at even lower flow rates, the amplitude being very much smaller. Figure 13 a and b have a time scale of 5 mm/sec. The time scale on Figure 13c is reduced to one mm/sec. and the voltage attenuation is increased in order to give a better overall picture.

A similar phenomenon to that illustrated in Figure 13 was also detected within the oil phase. The velocity fluctuations in the oil phase were, however, less violent than in the water phase. Further, the frequency of the sinusoidal motion

in the oil phase appeared to be slightly higher than that in the water phase; these being 0.6 and 0.54 cycles per second respectively. The frequencies were constant and independent of the respective fluid flow rates. The sinusoidal oscillations within the water at high oil rates appeared less pronounced than with low oil rates or with single phase water flow. No attempt was made to establish the factors which influence the frequencies of oscillations within each fluid. However, it is believed that the kinematic viscosity and conduit geometry are prime variables.

Figure 13 also illustrates the presence of a distinct "beating" of the sinusoidal velocity fluctuations. "Beating" occurs when two wavetrains of equal amplitude but slightly different frequencies travel through the same region. Interference by superposition of the two wavetrains results in fluctuations whose amplitude is not constant but varies in time.

Both, Figure 12 and Figure 13 show that high frequency fluctuations (3.5 cycles per second for the first and 9 cycles per second for the latter) are superimposed on the sinusoidal oscillations. These small fluctuations appear to have no significant effect on the transition to turbulences and are believed to be a result of the vibrations created by the liquid feed pumps. The high frequencies were dependent upon the valve settings and attempts to control these fluctuations were not successful. It should be noted that for very low Reynolds numbers ($N_{Re_w} = 260$) the high frequencies are not

present. At this low flow rate the main control valve to the test conduit was almost fully closed. Thus, fluctuations originating at the pumps could not readily be transmitted along the fluid across the valve.

The observations of the sinusoidal fluctuations seem to verify the theory of selective amplification and damping, and they are in agreement with the observations of Schubauer and Skramstad(34) who studied velocity fluctuations along an air boundary layer with low background turbulence intensity. The observations appear to support also Benney's(2) non-linear theory for oscillations in a parallel flow. The "beating" phenomenon described above appears to be an indication of the alternate partial reinforcement and cancellation of the secondary and primary vorticities described by Benney. The existence of secondary flow in the rectangular conduit used in this investigation was previously discussed by Sinha(35) in a thesis submitted to the University of Alberta. Measurements by Nikuradse(25) of constant velocity contours in a rectangular conduit also indicate the existence of such secondary flows.

Summarizing this phase of transition flow, small sinusoidal fluctuations appear even at very low flow rates. Their amplitude increases as the flow rate increases. Bursts of large amplitude oscillations develop which eventually break into spots of irregular, high-frequency turbulence.

2. Intermittent Flow

The second stage of transition to turbulence, in which neither fully laminar nor fully turbulent flow exists, was of primary interest in this work. The basic experimental data were obtained by placing the hot-film sensing element in the water phase at a constant distance from the conduit entrance, and recording the A.C. anemometer output signal for 800 seconds on the Dynograph recorder. For each run an oil rate was set and maintained constant while the water rate was increased in small increments over the entire transition range. Test runs were made for oil Reynolds numbers ranging from zero to 745. Dynograph records for each set of flow conditions were later measured and analysed to determine the degree to which turbulence had progressed.

A number of general observations on the nature of the intermittent turbulent fluctuations can be made. Figures 14 and 15 are typical of the hot-film records encountered at low ($0 \leq N_{Re_o} < 330$) and high ($N_{Re_o} > 330$) oil Reynolds numbers respectively. The nature of the intermittency in single-phase water flow was similar to that with low oil flow rates, i.e. Figure 14. The time scale in both figures is 1 mm/sec.

Figures 14 and 15 illustrate a distinct difference in the turbulent velocity fluctuations for low and high oil Reynolds numbers. At low oil Reynolds numbers the intermittent spots are not clearly defined and the fluctuating component frequencies

are relatively low. The amplitude of the fluctuations is very irregular. In the laminar sections of the intermittent flow pattern, a greater number of high amplitude fluctuations with frequencies of the same order of magnitude as the sinusoidal oscillations described earlier, are present. This may be an indication that under these flow conditions, amplification of selective frequencies above a critical amplitude is one of the important mechanisms resulting in turbulence transition. At high oil Reynolds numbers, i.e. $N_{Re_o} > 330$, the turbulent spots in the water phase are very sharply outlined and the fluctuating component frequencies appear to be much higher than at low oil Reynolds numbers. The amplitude of the fluctuations is relatively constant. The laminar sections of the intermittent flow pattern have only a few low frequency oscillations. In addition to the Tollmien-Schlichting instability mechanism other factors are therefore also believed to influence the transition to turbulence in the region of high oil Reynolds numbers.

During the initial stages of turbulent spot development in the water phase it was observed that closely spaced slugs of turbulence frequently passed the hot-film sensing element in groups of two, three or four. These slugs were preceded and followed by a relatively long interval of laminar flow. The time duration or length of the turbulent spots was approximately constant. The above observations suggest the possibility of the splitting process described by Lindgren(21)

taking place. He indicated that for pipe flow, in which intermittency is approaching an equilibrium value from below, there is a tendency for turbulent spots to grow to a definite length and then to divide rather than continue growing. Thus, one turbulent slug after being formed as a result of selected frequency amplification can create additional slugs by repeated splitting. This phenomenon appeared to be present for the entire range of oil flow rates investigated, but was most prominent at high oil rates. Figure 16 as well as Figures 14 and 15 show the features described above. Figure 16 also shows the D.C. voltage output from the anemometer. This signal is proportional to the time average velocity of the water flowing past the hot film. The average water velocity decreases as the turbulent patches pass the observation point. The difference between the average laminar and turbulent velocity is less pronounced for low, $N_{Re_o} < 330$, Reynolds numbers. Two parameters were defined and used in the quantitative analysis of the hot-film anemometer output records in this study. An intermittency number ($\bar{n}$) and an intermittency factor (γ) were determined for each set of flow conditions. The intermittency number expresses the average number of changes from laminar to turbulent flow per unit time. It is equal to the number of changes which have occurred divided by the total time of observation. The intermittency factor indicates the degree to which turbulence has progressed and is defined as the sum of the time intervals during which turbulent flow exists, divided

by the total observation time.

Figures 17, 18 and 19 illustrate the relationship between the water intermittency number and the water Reynolds number for a series of oil rates flowing co-currently with the water in the channel. The intermittency numbers were made dimensionless with the equivalent diameter of the conduit and the average superficial velocity. The intermittency factors corresponding to the above flow rates were plotted versus the water Reynolds number in Figures 20, 21, and 22.

The S-shaped curves of Figures 20, 21 and 22 suggest a normal Gaussian distribution of the intermittency factor. The data were plotted again on probability paper and are presented in Figures 23, 24, and 25.

The data of this investigation may be subject to some error. The volume of data collected is insufficient for a statistical analysis. In addition, a certain degree of judgement, which is always subject to error, was required in marking and measuring the transition data. The intermittency data for all water flow rates were obtained from anemometer output records of uniform length. Thus, the number of turbulent slugs found on any one trace was dependent upon the degree of intermittency. Both at low and at high intermittency factors the number of spots in the sample used to describe the average condition of the flow was much less than for intermittency in the 0.5 range. The reliability of the experimental intermittency numbers and factors is, therefore, highest in the

central transition region. The possible lack of statistical accuracy in the high and low intermittency zones accounts for the deviation of the γ factors from the straight line probability curve in Figures 23, 24 and 25.

Additional meaning can be given to the intermittency number and factor if these values are expressed in terms of an average time required for alternate laminar and turbulent patches to pass the point of observation. The inverse of the intermittency number represents the average total time for the passage of one complete laminar-turbulent cycle. This average time, made dimensionless by the average superficial water velocity and the equivalent diameter of the conduit, is given for several oil flow rates, by the parabolic curves of Figure 26. Further, the average duration time of turbulent slugs can be obtained by applying a factor of γ to the average total cycle time. This curve is also shown on Figure 26. The difference between the two curves represents the average duration time of the laminar regimes.

Total dimensionless time of laminar-turbulent cycle

$$= \frac{\bar{u}}{\bar{n}d} = \frac{\bar{t}u}{d}$$

Dimensionless time of turbulent slugs

$$= \frac{\bar{u}}{\bar{n}d} \gamma = \frac{\bar{t}_T \bar{u}}{d}$$

Dimensionless time of laminar slugs

$$= \frac{\bar{u}}{\bar{n}d} (1 - \gamma) = \frac{\bar{t}_L \bar{u}}{d}$$

The data for Figure 26 were obtained from the corresponding smoothed curves for intermittency numbers and intermittency factors.

Some of the more interesting features of Figures 17 to 26 require discussion in greater detail. These figures give an overall insight to the effect that the changing oil flow rate has on the transition to turbulence in the water phase. As transition to turbulence progresses due to increases in the water flow rate, the intermittency number at a specific oil rate increases from zero to a maximum and then returns again to zero (Figures 17, 18, 19). In general, the maximum occurs at or near that water Reynolds number for which the intermittency factor is equal to 0.5; i.e. when laminar and turbulent flow exist for approximately equal time periods. This implies that in the first half of the transition period, the degree of turbulence is increased due to the formation of additional turbulent slugs, whereas in the second half of the transition period, the degree of turbulence is increased by the expanding growth and melting together of turbulent spots.

Figures 17 to 26 indicate that the water Reynolds number at which transition occurs is strongly dependent upon the Reynolds number of oil flowing co-currently with the water. The curves for specific oil rates are shifted laterally along the axis for the water Reynolds number. In order to show this dependence more clearly, a cross-plot of the oil Reynolds

number versus the transition water Reynolds number was made and is presented in Figure 27. As transition to turbulence does not occur instantaneously the transition water Reynolds number is arbitrarily associated with a particular intermittency factor. Transition water Reynolds numbers at γ equal to 0.2, 0.5 and 0.8 were plotted. The vertical broken lines correspond to transition in water for zero oil flow. Data for higher and lower intermittency factors than those plotted are believed to be subject to large error and are not included in Figure 27. Also presented in Figure 27 are the loci of the critical water Reynolds number as calculated by the semi-empirical stability parameter.

Figure 27 illustrates an extremely interesting phenomenon in fluid flow. There appears to be an anomaly in the loci of the transition water Reynolds numbers at N_{Re_o} approximately equal 330. The existence of this anomaly is supported by a number of independent observations. It should be pointed out that the experimental data of Figure 27 were not obtained in consecutive order, therefore, eliminating the possibility that a trend in experimental error was followed. Attention should also be given to the plots of the intermittency numbers and factors. Of particular interest are the curves for N_{Re_o} equal to 320 and 353. These curves yield the data points which bracket the anomaly on both sides. The intermittency number plot at $N_{Re_o} = 320$, again in Figure 19, shows two maxima. This indicates that a lateral shift in the transition water

Reynolds number has occurred. The lateral shift is also shown in the corresponding intermittency factor curve (Figure 20 and 22). The intermittency factor plot for $N_{Re_o} = 320$ follows the expected S-shaped curve for γ less than 0.7; however, at γ equal to 0.75 the curve suddenly shifts laterally to a higher value of N_{Re_w} . Figure 24 again shows this lateral shift. The change in position of the curves described above occurs at the same water Reynolds number as the anomaly in Figure 27. Figures 18 and 21 present the intermittency numbers and factors respectively, for $N_{Re_o} = 353$. Transition to turbulence in the water phase for this oil rate occurs over a much wider range of N_{Re_w} than at higher and lower oil rates. The features described above are in agreement with the anomaly in Figure 27.

Rotta's(29) experimental presentation of the development of turbulent flow in a pipe revealed a functional dependence of the intermittency number and factor upon the pipe entrance length as well as upon the Reynolds number. The entrance length was expressed in terms of a length to diameter ratio (x/d). He found that for increasing values of x/d , the range of Reynolds numbers in which the flow was intermittent narrowed and shifted to a lower value of N_{Re} . The intermittency number became progressively smaller with increased pipe length. However, the lower critical Reynolds number remained constant; only the rate of transition changed, i.e. the curves of γ versus N_{Re} became steeper with increased pipe length.

In two-phase flow, the addition of the second phase essentially reduces the effective diameter of the conduit with respect to the first phase. Thus, increasing the oil flow rate increases the effective x/d ratio; which, in turn, should result in a decrease in the width of the transition region. The lower critical Reynolds number should remain constant. Figures 23, 24 and 25 suggest that the above conditions are not met. For oil Reynolds numbers less than 300, the intermittency factors, plotted on probability paper, form a family of parallel straight lines. The slope of the curves ($N_{Re_O} < 300$) is equal to the slope for single phase water flow. The parallel laterally spaced lines imply that for these oil flow rates, the width of the water transition region is constant and that the lower critical water Reynolds number is a function of N_{Re_O} . For oil Reynolds numbers greater than 500, a family of curves having a steeper slope than for N_{Re_O} less than 300, appears to exist. The range of N_{Re_w} , during which flow is intermittent, is not as wide as for N_{Re_O} less than 500. Curves for N_{Re_O} equal to 320, 353 and 417 do not fit into either of the above families and are believed to be the result of the anomaly mentioned earlier. The curve for N_{Re_O} equal to 465 again has the same slope as curves for N_{Re_O} less than 300. From the above observations it is concluded that the change in effective conduit diameter due to the addition of the oil, is not the primary factor affecting the transition to turbulence in the water phase.

Figure 27 suggests that the water transition Reynolds number can be divided into three regions dependent upon the Reynolds number of the oil flowing co-currently with the water, and according to the factors which determine the energy transfer from the mean flow to the fluctuating velocity components within the water phase. For the system under investigation, these regions are defined as follows:

Region 1: for oil Reynolds numbers less than 35;

Region 2: for oil Reynolds numbers greater than 35 and less than 330;

Region 3: for oil Reynolds numbers greater than 330.

The regions are delineated on Figure 27 by the horizontal broken lines and are believed to extend over the range of water Reynolds numbers for which the flow is intermittent.

The basic mechanism of instability is believed to be the same for the entire range of oil Reynolds numbers considered. However, different elements in the different regions are superimposed and influence the basic mechanism. This mechanism is believed to akin to the Tollmien-Schlichting instability described earlier. One basic mechanism is suggested by the fact that for the whole range of oil Reynolds numbers tested, velocity fluctuations were most violent near the conduit wall; implying that the point of greatest instability was also near the wall. Furthermore, sinusoidal fluctuations implying selective amplification of disturbances, were present in the water

for all oil flow rates tested.

From Figure 27 it is evident that for oil Reynolds numbers below 35, the transition to turbulence in the water phase, in the presence of the oil, occurs at a higher water Reynolds number than for single phase water flow. The thin oil layer appears to have a stabilizing effect on the water. Energy from velocity fluctuations in the water is thought to be absorbed and dissipated in the compliant oil layer.

As mentioned earlier, a similar stabilizing effect was observed by Charles(4) and Gazley(14). Essentially, the same basic flow equipment was used in the present investigation as by Charles. The fluid properties were only slightly different in the two investigations. Charles observed the breakup of injected dye filaments to determine the transition Reynolds numbers. The stabilizing effect noted in the present investigation was not as pronounced as that indicated by Charles in Figure 1. Figure 1 shows that transition to turbulence in the water is delayed for oil Reynolds numbers less than 300; whereas Figure 27 shows this delay as occurring for oil Reynolds numbers less than 35.

The experimental transition water Reynolds numbers in Region 2 ($35 < N_{Re_o} < 330$) are in good agreement with the calculated values described in Section III for the lower critical water Reynolds number for the rectangular conduit. The calculations for the critical Reynolds numbers do not take into account any interfacial effects, other than the decrease of the equiva-

lent conduit diameter due to the second phase. The oil phase, therefore, appears to have little effect upon the turbulence transition in Region 2. The turbulence structure was similar for Regions 1 and 2 as illustrated in Figure 14.

For oil Reynolds numbers less than 300 and for water flow rates slightly above the upper critical water Reynolds number the intermittency factor was unity; however, the turbulence structure, as observed on the anemometer output records was not uniform. Patches of high frequency, high amplitude fluctuations alternated with sections of random low frequency, low amplitude fluctuations. The turbulence structure became more and more uniform as the water Reynolds number was increased above the upper critical value.

The lower boundary of Region 3, as illustrated in Figure 27, is set at the oil flow rate at which a small increase in the oil Reynolds number results in a relatively large increase in the critical transition water Reynolds number.

It is postulated that the anomaly in the locus of the transition water Reynolds number, shown in Figure 27 at $N_{Re_o} = 330$, occurs as a result of the position of the maximum velocity with respect to the location of the interface. Calculated results, as presented on Figure 27, as well as the experimental measurements outlined in Table 22, indicate that the anomaly occurs at an interface position approximately equal to 0.38. The interface position is the fractional distance of the total plate separation from the bottom plate of the conduit. For interface

positions less than 0.38 turbulence transition in the water phase is delayed to a higher water Reynolds number. This delay is believed to be a direct consequence of the change in the velocity distribution accompanying the change from a fully laminar to a fully turbulent flow pattern in the lower water phase.

Figure 7 presents the theoretical laminar velocity distribution at the central vertical cross-section of the rectangular conduit. It can be seen that as the oil-water interface is lowered, due to increased oil-water flow ratios, the maximum velocity which occurs initially in the water phase, approaches the interfacial velocity. At interface position 0.3 the maximum velocity occurs at the interface. A further increase in the oil-water ratio causes the maximum velocity to shift from the water phase to the oil phase. During intermittent flow in the water phase, the velocity distribution alternates between the fully laminar and fully turbulent velocity profiles. In turbulent flow, the momentum transfer perpendicular to the flow direction results in a lower average as well as a lower maximum velocity in the water phase. For interface positions greater than about 0.4 the above velocity changes merely result in a slight increase of approximately 0.01 to 0.015 inches in the thickness of the water layer as a turbulent spot passes. The interfacial velocity is not affected significantly. However, as the interface position becomes less

than about 0.38 the change to turbulent flow has a pronounced effect on the oil phase. Under these conditions, as the maximum water velocity is near or at the interface, the turbulent momentum transfer in the water layer will tend to reduce the interfacial velocity and consequently alter the velocity distribution of the laminar oil layer as well as that of the water layer. The new velocity distribution will result in a higher rate of shear. To bring about and maintain turbulence, the energy level of the water phase must also be higher under these conditions than if little or no change in the interfacial velocity is necessary. The increased internal energy is reflected in the sudden increase in the critical transition water Reynolds number at $N_{Re_o} = 330$ as shown in Figure 27. The presence of this additional internal energy also becomes evident in comparing the frequencies of the turbulent velocity fluctuations in Figures 14 and 15. The velocity fluctuation frequency in the water phase is much lower for N_{Re_o} less than 330 than for N_{Re_o} greater than 330. Further, Figure 18 and Figure 27 indicate that the duration time of a laminar-turbulent cycle is higher for N_{Re_o} greater than 330, than for N_{Re_o} less than 330. This suggests greater stability of the intermittent flow pattern at high oil Reynolds numbers.

Attempts were also made in this investigation to determine the dependence of the intermittency of flow upon the position (distance from the lower plate) of the sensing element.

The variations in the physical properties of the fluids occurring as a result of the gradual changes in the fluid temperatures with respect to time, presented a problem. Although constant volumetric flow rates could be maintained for extended periods of time, the Reynolds numbers did not remain constant. For zero oil Reynolds numbers, the sensing element was set at a particular position and the water flow rate was increased incrementally over the transition region. The sensing element was then raised to another position and the water flow rates were again varied incrementally. A record of the anemometer output signal was obtained for each flow rate and an intermittency number and factor were measured and calculated. The Reynolds numbers were calculated for each flow increment using the physical properties of the fluids at the existing fluid temperatures. The resulting intermittency numbers and factors for four probe positions are given in Figure 19 and 22. For the two-phase flow system, both the oil and water volumetric flow rates were maintained constant. The hot-film probe was placed in position and the anemometer output signal was recorded for 800 seconds. The hot-film sensing element was then placed in the second position and the anemometer signal was recorded for 1600 seconds. Next, the probe was again returned to the first position and the output signal was recorded for another 800 seconds. Intermittency numbers and factors were determined from the output traces for each probe position. The changes in the fluid properties due to the gradually increasing temperature were assumed linear, therefore, the average values of $\bar{n}$ and γ as obtained above correspond

to the Reynolds numbers calculated using average fluid properties.

The variation of the intermittency with respect to the sensing element position was investigated in the above manner for oil Reynolds numbers equal 65, 450 and 632. The water phase in each case was in the intermittent flow regime. The results for these two-phase experiments are presented in Table 23. Figures 19 and 22 and Table 23 indicate that, within the limitations of the equipment and experimental error of this investigation, the degree of intermittency in the water phase is constant across the vertical cross-section of the conduit. This implies that, at the downstream position where the intermittency measurements were made, the turbulent patches had developed sufficiently to occupy the entire water phase. Presumably, however, there must be a point close to the entrance section where the turbulent spots originate and, therefore, do not occupy the full cross-section of the fluid phase. Further, a thin fully laminar sublayer will exist even in regions of fully developed intermittent flow.

3. Interfacial Wave Structure

The appearance of interfacial waves coincided in each case with the appearance of the turbulent patches in the water phase. The wave structure, however, changed with increased oil flow rates. At low oil rates, the passage of a turbulent spot

was marked by an increase in thickness of the water layer of 0.010 to 0.015 inches. These waves of low amplitude and long wavelength were a result of the decreased average water velocity of the turbulent patches. A single "wave" of this nature existed for each turbulent spot. Superimposed on these waves were extremely minute ripples or disturbances. Each individual disturbance appeared to be caused by a discrete fluid packet of high momentum, travelling perpendicular to the direction of flow, and hitting the oil-water interface.

At oil Reynolds numbers approaching and exceeding 300 the interfacial wave action corresponding to each turbulent spot became more and more pronounced; resulting in the formation of small two-dimensional waves. The waves had crests two to three times their wavelength and extended over one-third to one-half of the conduit width. A number of these waves existed for each turbulent spot. The number increased with increased length of the turbulent spot. The wave action became continuous as the flow became fully turbulent. It is believed that the formation of these waves is strongly influenced by the interfacial shear and the velocity differences near the interface of the two fluids.

It is interesting to note that a spot check on intermittent flow in the oil phase at N_{Re_o} equal 1200 and N_{Re_w} equal 1560 resulted in interfacial waves similar to those described above. The wave patches in this case correspond to

intermittent turbulent spots within the oil phase. Only a study considering simultaneously the transition to turbulence in both the oil and water phases, could determine if the turbulent spots originate

1. in the water phase and are transmitted across the interface by the wave action to the oil phase, or
2. in the oil phase, influenced by a laminar water layer.

Possibility 1 above is considered the most probable as the flow rates were in line with the transition locus of Figure 27.

VII. CONCLUSIONS

1. Within limited regions the lower critical Reynolds numbers for laminar-turbulent transition as calculated theoretically for the two-phase oil and water system are in agreement with experimental values. Agreement is good for critical water Reynolds numbers at oil Reynolds numbers less than 330 and for critical oil Reynolds numbers at water Reynolds numbers less than 1000. Under these flow conditions, interfacial interaction is small.
2. The instability of steady laminar motion in the single- and two-phase liquid systems leads to regular sinusoidal oscillations or secondary flow. The oscillations appear even at very low Reynolds numbers. Their amplitude increases with increasing flow rates; their frequency, however, remains constant. The basic mechanism of instability appears to be that of selective amplification of small disturbances.
3. Bursts of large amplitude oscillations develop in the water phase and eventually break into spots of irregular, disturbed flow. The flow field then alternates between a laminar and a turbulent flow pattern. The number of laminar-turbulent cycles increases for the first half and decreases for the second half of the transition period.
4. The intermittency factor for the water phase as a function of the water Reynolds number, appears to follow a Gaussian distribution.

5. The water transition Reynolds number can be divided into three regions. In Region 1, for oil Reynolds numbers less than 35, transition to turbulence in the water phase occurs at a higher water Reynolds number than for single-phase water flow. The thin oil layer tends to stabilize the water phase. In Region 2, for oil Reynolds numbers greater than 35 and less than 330, the oil phase appears to have little or no effect, other than the change in effective conduit diameter, on water transition flow. In Region 3, $N_{Re_o} > 330$, a sudden increase in the transition water Reynolds number occurs. A comparison of velocity fluctuation frequencies suggests that turbulent spots for Region 3 have a higher internal energy level than spots in Regions 1 and 2.
6. At the downstream position of intermittency measurements and within the limitations of this investigation, the intermittency factor for the water phase is constant across the vertical cross-section of the conduit.
7. For the flow rates investigated, the appearance of interfacial waves coincides with the appearance of turbulent spots. The interfacial wave action becomes more pronounced as the oil Reynolds number is increased, resulting in small two-dimensional waves for N_{Re_o} exceeding 300.

VIII. RECOMMENDATIONS

1. The stability parameter K should be modified to incorporate the effects of gravitational forces, interfacial tension and shear as well as the density and viscosity ratios on the two-phase system.
2. Experimental equipment to be used in extending this investigation should include an automatic temperature control system capable of maintaining the liquids within $\pm 0.1^{\circ}\text{F}$. The equipment should also include a mechanism to permit rapid calibration of the hot-film anemometer in order to obtain quantitative velocity measurements.
3. Detailed experimental results on the sinusoidal oscillations or secondary motion would be of value. Attempts should be made to relate spanwise variation and interaction, amplitude growth and frequency variation in single- and two-phase systems to the flow parameters.
4. Further information on the growth rate in the spanwise and downstream direction of turbulent spots in single- and two-phase flow would be desirable in establishing the transition mechanism.
5. The interfacial velocity would be a valuable measurement to show the influence of the oil phase on turbulence transition in the water phase and vice-versa. The interfacial velocity would also help to show the relationship between the inception of turbulent spots and that of small two-dimensional interfacial waves.

6. A comparative study of interfacial fluctuations and turbulent fluctuations would help to resolve the relationship between these phenomena. Power spectra for interfacial fluctuations obtained by using a photometric technique(37) could be compared to power spectra for turbulent fluctuations obtained from a hot-film anemometer output signal.
7. A simultaneous study of the intermittent flow pattern for oil and water Reynolds numbers less than 3000, would further show the interaction between the fluid phases. It would be interesting to see if an anomaly such as observed for the transition water Reynolds number at $N_{Re} = 330$, also exists for the oil phase. An effort could also be made to verify the existence and establish the importance of the splitting phenomenon of turbulent spots in laminar-turbulent transition flow.

IX. NOMENCLATURE

a	half width of the rectangular conduit, (ft)
A	defined as $(\mu_1 b_2^2 - \mu_2 b_1^2) / (\mu_1 b_2 + \mu_2 b_1)$, (ft)
b_i	depth of the fluid layer i, (ft)
d	conduit diameter or characteristic length, (ft)
d_e	equivalent diameter equal 4 times the hydraulic radius, (ft)
F	body forces per unit mass
g	acceleration due to gravity, (ft/sec ²)
g_c	gravitational constant, (lb _m ft/lb _f sec ²)
h	half separation of parallel plates, (ft)
h_i	half separation of hypothetical parallel plates for fluid i, (ft)
H	actual parallel plate separation, $H = h_1 + h_2$, (ft)
J	Richardson number, $J = -(g \, d\rho/dy) / \rho (du/dy)^2$
K	stability parameter, $K = \rho \text{grad}(\vec{u} \cdot \vec{u}) / (2 F - \text{grad } P)$
K_c	critical stability parameter, $K_c = 404$
$\bar{n}$	intermittency number; average number of changes from laminar to turbulent flow per second
N_{Re_o}	superficial oil Reynolds number based on total height of conduit
N_{Re_w}	superficial water Reynolds number based on total height of conduit
P	pressure, (lb _f /ft ²)
q_i	volumetric flow rate, (USG/min)
Q_i	volumetric flow rate, (ft ³ /sec)

r_w	pipe radius, (ft)
u	point velocity of fluid, (ft/sec)
u_b	average velocity of fluid, (ft/sec)
x, y, z	rectangular coordinates, (ft)
$\bar{y}_i$	position where K_i parameter is maximum, (ft)
γ	intermittency factor; time during which flow is turbulent divided by total observation time
μ_i	viscosity of fluid i, (lb_m /ft.sec)
ν	kinematic viscosity, (μ/ρ , ft^2 /sec)
ρ	density, (lb_m /ft ³)
$\bar{t}$	total average duration time of laminar-turbulent cycle, (sec)
$\bar{t}_T$	average duration time of turbulent spot, (sec)

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APPENDIX A

TABLES

The physical properties of the liquids used in the theoretical calculations and the experimental investigation are described in Table 1. Table 2 gives theoretical transition oil and water Reynolds numbers for parallel plates and rectangular conduit which are plotted in Figure 6. The data in Table 3 for the root-mean-square voltage as a function of average velocity is shown in Figure 11. Table 4 to 7 represent experimental data for flow of water alone. Table 8 to 21, representing the data for the two-phase system, are subdivided according to the superficial oil Reynolds number, N_{Re_o} . The results from Tables 4 to 21 are shown in Figure 17 to 25. The data from Table 24 and Table 22 are presented in Figures 26 and 27 respectively. Table 23 gives the intermittency number and factor of water, in two-phase flow, at different probe positions. Oil Reynolds numbers were calculated using the physical properties at the mean oil temperature for the experimental run. Water Reynolds numbers were calculated using the physical properties corresponding to the actual temperatures at each individual flow rate. During an experimental run, water temperatures deviated by up to $\pm 2^{\circ}\text{F}$ from the mean values quoted. Measurements were taken 24 feet or 162 equivalent diameters from the conduit entrance. Pressure measurements were recorded in centimeters of monometer fluid per 20 feet of conduit. The density of the monometer fluid was 1.0994 gm/cc.

Table 1
Physical Properties of Liquids

Temp. $\pm 0.02^{\circ}\text{F}$	Water			Oil		
	Density gm/cc	Viscosity cp	$\frac{N_{Re_w}}{q_w}$	Density gm/cc	Viscosity cp	$\frac{N_{Re_o}}{q_o}$
75	0.99603	0.9277	590.1	0.82053	5.3360	84.5
78	0.99490	0.8909	613.8	0.82065	5.0819	88.8
81	0.99416	0.8570	637.6	0.81878	4.7840	94.1
84	0.99359	0.8256	661.5	0.81760	4.6211	97.2
87	0.99279	0.7959	685.6	0.81639	4.4093	101.8
90	0.99217	0.7688	709.3	0.81477	4.2141	106.3

Table 2

Theoretical Transition Oil and Water Reynolds Numbers for Parallel

Plates and Rectangular Conduit

Interface Position (Fraction Water)	Parallel Plates				Rectangular Conduit			
	Transition in Water Phase N_{Re1}^*	Transition in Oil Phase N_{Re1}	Transition in Oil Phase N_{Re2}	Transition in Water Phase N_{Re1}	Transition in Water Phase N_{Re2}	Transition in Oil Phase N_{Re1}	Transition in Oil Phase N_{Re2}	
0.0	-	-	0	2,799	-	0	2,293	
0.1	149	331	986	2,197	119	792	1,753	
0.2	694	511	2,343	1,724	554	1,890	1,372	
0.3	1,424	483	4,097	1,389	1,162	3,343	1,111	
0.4	2,030	342	6,611	1,115	1,675	5,459	900	
0.5	2,417	201	10,424	868	2,011	8,666	707	
0.6	2,629	102	16,384	637	2,192	13,661	524	
0.7	2,732	44	25,862	419	2,276	21,536	347	
0.8	2,778	15	41,136	221	2,306	34,224	185	
0.9	2,795	3	66,073	67	2,304	55,760	58	
1.0	2,799	0	-	-	2,293	-	-	

* 1 - Water
2 - Oil

Table 3

Root-Mean-Square Voltage as a
Function of Average Velocity

Average Water Temperature = 78°F

N_{Re_w}	$\bar{u}_w$ (ft/sec)	$\Delta P/\Delta L$ (cm of fluid)	R.M.S.V. (Volts)
505	0.0330	-	0.150
1,003	0.0656	0.115	0.163
1,200	0.0785	0.145	0.130
1,484	0.0970	0.190	0.308
1,716	0.1120	0.365	0.245
1,995	0.1305	0.395	0.275
2,140	0.1400	0.525	0.435
2,445	0.1598	0.490	0.510
2,985	0.1950	0.695	0.90
3,530	0.231	1.560	1.15
3,670	0.240	1.640	1.05
5,150	0.336	2.735	1.20
7,070	0.462	4.670	1.30
9,200	0.602	7.930	1.34
13,700	0.895	17.785	1.55

Table 4Run No. 1

Mean water temperature = 81.5°F

Distance of probe from lower plate = 0.180 inches

Oil flow rate = 0

No.	Water Flow Rate (USGPM)	N_{Re_w}	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{u}$
1	3.04	1,930		0	0	0
2	3.42	2,170		0	0	0
3	3.82	2,425		0.016	0.0024	0.034
4	3.92	2,510	0.560	0.090	0.0112	0.164
5	4.01	2,560	0.660	0.129	0.0168	0.252
6	4.21	2,685	0.685	0.411	0.0339	0.534
7	4.31	2,760	0.735	0.749	0.0234	0.377
8	4.41	2,840	0.690	0.869	0.0216	0.356
9	4.61	2,980	0.800	0.970	0.0106	0.183
10	5.36	3,470	1.125	1	0	0

Table 5
Run No. 2

Mean water temperature = 82.0°F

Distance of probe from lower plate = 0.287 inches

Oil flow rate = 0

No.	Water Flow Rate (USGPM)	N_{Re_w}	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	3.33	2,130	0	0	0
2	3.62	2,320	0	0	0
3	3.82	2,450	0.040	0.0068	0.097
4	3.92	2,520	0.118	0.0129	0.189
5	4.01	2,590	0.171	0.0194	0.291
6	4.11	2,660	0.462	0.0307	0.472
7	4.21	2,730	0.660	0.0289	0.455
8	4.31	2,800	0.787	0.0228	0.368
9	4.41	2,870	0.947	0.0130	0.214

Table 6Run No. 3

Mean water temperature = 83.0°F

Distance of probe from lower plate = 0.393 inches

Oil flow rate = 0

No.	Water Flow Rate (USGPM)	N_{Re_w}	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{u}$
1	3.33	2,190	0	0	0
2	3.62	2,365	0	0	0
3	3.92	2,570	0.227	0.0210	0.308
4	4.01	2,625	0.397	0.0339	0.509
5	4.11	2,690	0.407	0.0280	0.430
6	4.41	2,720	0.487	0.0438	0.724
7	4.21	2,750	0.761	0.0262	0.412
8	4.31	2,810	0.845	0.0178	0.287
9	4.61	2,850	0.832	0.0270	0.466

Table 7Run No. 4

Mean water temperature = 82.0°F

Distance of probe from lower plate = 0.500 inches

Oil flow rate = 0

No.	Water Flow Rate (USGPM)	N_{Re_w}	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{nd}}{\bar{u}}$
1	3.33	2,150	0	0	0
2	3.62	2,340	0	0	0
3	3.82	2,460	0.064	0.0038	0.054
4	3.92	2,530	0.199	0.0200	0.292
5	4.01	2,590	0.278	0.0240	0.360
6	4.11	2,655	0.494	0.0302	0.464
7	4.21	2,710	0.611	0.0285	0.449
8	4.31	2,790	0.855	0.0197	0.318
9	4.41	2,840	0.894	0.0202	0.333

Table 8Run No. 5

Mean water temperature = 83.0°F

Mean oil temperature = 83.5°F

Oil flow rate = 0.250 USGPM

 $N_{Re_o} = 26$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	3.04	1,970	0.731	0	0	0
1	3.52	2,280	0.747	0	0	0
2	3.92	2,570	0.760	0.063	0.0100	0.146
3	4.01	2,620	0.760	0.113	0.0162	0.243
4	4.21	2,750	0.770	0.338	0.0318	0.501
5	4.31	2,820	0.770	0.586	0.0415	0.669
6	4.41	2,900	0.775	0.830	0.0282	0.465
7	4.61	3,030	0.785	0.964	0.0060	0.103

Oil flow rate = 0.388 USGPM

 $N_{Re_o} = 38$

8	4.01	2,640	0.758	0.418	0.0398	0.597
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Table 9Run No. 6

Mean water temperature = 86.0°F

Mean oil temperature = 88.0°F

Oil flow rate = 0.538 USGPM

 $N_{Re_o} = 58$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	1.52	1,005	0.540	1.235	0	0	0
2	2.29	1,535	0.602	1.320	0	0	0
3	3.13	2,120	0.625	1.455	0.031	0.0086	0.1007
4	3.23	2,190	0.626	1.465	0.039	0.0093	0.1123
5	3.47	2,385	0.630	1.570	0.210	0.0378	0.491
6	3.62	2,455	0.637	1.610	0.553	0.0492	0.666
7	3.62	2,493	0.635	1.565	0.625	0.0565	0.765
8	3.83	2,595	0.640	1.770	0.848	0.0268	0.383
9	4.01	2,730	0.648	1.780	0.927	0.0110	0.165
10	4.21	2,880	0.650	1.865	0.926		
11	4.41	3,020	0.655		0.965		
12	4.61	3,160	0.660		1	0	0

Table 10Run No. 7

Mean water temperature = 84.0°F

Mean oil temperature = 80.5°F

Oil flow rate = 0.642 USGPM

 $N_{Re_o} = 61$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	3.23	2,125	0.639	2.140	0	0	0
2	3.42	2,250	0.645	2.180	0	0	0
3	3.62	2,390	0.654	2.290	0.110	0.0170	0.230
4	3.72	2,460	0.657	2.315	0.345	0.0402	0.560
5	3.82	2,520	0.663	2.355	0.584	0.0496	0.708
6	3.92	2,600	0.670	2.530	0.853	0.0294	0.431
7	4.01	2,660	0.676	2.500	0.921	0.0171	0.257
8	4.21	2,800	0.636	2.630	0.968	0.0123	0.194

Table 11Run No. 8

Mean water temperature = 82.5°F

Mean oil temperature = 84.0°F

Oil flow rate = 0.784 USGPM

 $N_{Re_o} = 76$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{u}$
1	3.42	2,200	0.613	1.625	0	0	0
2	3.62	2,325	0.619	1.640	0	0	0
3	3.82	2,460	0.631	1.705	0.288	0.0432	0.617
4	3.92	2,540	0.637	1.770	0.600	0.0594	0.870
5	4.01	2,600	0.643	1.800	0.809	0.0474	0.711
6	4.21	2,750	0.650	1.990	0.895	0.0277	0.437
7	4.41	2,880	0.630	2.065	1	0	0
8	4.61	3,010	0.630	2.115	1	0	0

Table 12Run No. 9

Mean water temperature = 83.5°F

Mean oil temperature = 82.0°F

Oil flow rate = 1.113 USGPM

 $N_{Re_o} = 105$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	3.04	2,000	0.562	1.870	0	0	0
2	3.23	2,125	0.571	1.905	0	0	0
3	3.42	2,250	0.574	1.940	0.047	0.0070	0.089
4	3.62	2,380	0.585	1.960	0.219	0.0336	0.455
5	3.82	2,510	0.600	2.005	0.617	0.0555	0.793
6	4.01	2,650	0.602	2.085	0.844	0.0281	0.422
7	4.21	2,780	0.612	2.175	0.907	0.0240	0.380
8	4.41	2,915	0.617	2.300	0.937	0.0186	0.325
9	4.61	3,040	0.623	2.390	1	0	0

Table 13Run No. 10

Mean water temperature = 83.5°F

Mean oil temperature = 81.5°F

Oil flow rate = 1.85 USGPM

 $N_{Re_o} = 174$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	2.29	1,495	0.445	2.260	0	0	0
2	3.04	1,990	0.484	2.355	0	0	0
3	3.13	2,065	0.489	2.560	0.070	0.0123	0.144
4	3.23	2,120	0.494	2.375	0.143	0.0243	0.293
5	3.23	2,140	0.494	2.680	0.127	0.0235	0.284
6	3.42	2,240	0.503	2.430	0.461	0.0453	0.579
7	3.62	2,370	0.512	2.645	0.769	0.0555	0.751
8	3.82	2,500	0.526	2.750	0.930	0.0276	0.394
9	4.01	2,640	0.530	2.785	0.960	0.0238	0.357
10	4.21	2,760	0.538	2.845	1	0	0
11	4.41	2,900	0.540	2.930	1	0	0

Table 14Run No. 11

Mean water temperature = 83.5°F

Mean oil temperature = 81.0°F

Oil flow rate = 2.51 USGPM

 $N_{Re_o} = 234$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{u}$
1	1.52	985	0.344	2.770	0	0	0
2	2.24	1,460	0.399	2.900	0.011	0.0012	0.010
3	2.44	1,600	0.408	2.945	0.034	0.0070	0.064
4	2.64	1,735	0.417	3.145	0.132	0.0200	0.197
5	2.84	1,870	0.427	3.190	0.263	0.0396	0.421
6	3.04	2,000	0.441	3.255	0.533	0.0434	0.494
7	3.13	2,060	0.443	3.310	0.705	0.0365	0.416
8	3.23	2,125	0.453	3.330	0.821	0.0288	0.348
9	3.42	2,250	0.460	3.445	0.981	0.0076	0.097
10	3.62	2,380	0.469	3.580	1	0	0
11	3.82	2,515	0.471	3.625	1	0	0

Table 15Run No. 12

Mean water temperature = 84.5°F

Mean oil temperature = 81.0°F

Oil flow rate = 3.15 USGPM

 $N_{Re_o} = 293$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	1.52	1,000	0.312	2.645	0	0	0
2	2.29	1,490	0.368	2.760	0.046	0.0088	0.075
3	2.44	1,620	0.373	2.800	0.160	0.0215	0.196
4	2.54	1,690	0.378	2.815	0.274	0.0295	0.280
5	2.64	1,750	0.386	2.885	0.483	0.0430	0.424
6	2.74	1,825	0.392	2.890	0.644	0.0452	0.464
7	2.84	1,890	0.396	2.950	0.778	0.0274	0.291
8	3.04	2,025	0.404	3.060	0.884	0.0198	0.225
9	3.23	2,150	0.412	3.170	0.925	0.0154	0.186
10	3.42	2,280	0.417	3.300	1	0	0
11	3.62	2,410	0.425	3.430	1	0	0

Table 16Run No. 13

Mean water temperature = 83.0°F

Mean oil temperature = 84.5°F

Oil flow rate = 3.27 USGPM

 $N_{Re_o} = 320$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	2.04	1,300	0.350	2.040	0	0	0
2	2.24	1,450	0.353		0.011	0.0018	0.015
3	2.54	1,650	0.360	2.125	0.206	0.0282	0.268
4	2.74	1,780	0.375	2.180	0.567	0.0376	0.385
5	2.84	1,870	0.380	2.300	0.715	0.0296	0.315
6	2.94	1,910	0.384	2.290	0.755	0.0290	0.319
7	3.13	2,050	0.397	2.350	0.753	0.0315	0.369
8	3.33	2,180	0.405		0.815	0.0196	0.244
9	3.52	2,310	0.409	2.610	0.977	0.0034	0.045

Table 17Run No. 14

Mean water temperature = 81.5°F

Mean oil temperature = 82.5°F

Oil flow rate = 3.70 USGPM

 $N_{Re_o} = 353$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	2.29	1,440	0.334	1.875	0	0	0
2	2.44	1,540	0.343	2.075	0.067	0.0134	0.122
3	2.54	1,610	0.343	2.155	0.075	0.0119	0.113
4	2.64	1,670	0.348	2.235	0.131	0.0172	0.170
5	2.74	1,740	0.351	2.300	0.131	0.0122	0.125
6	2.84	1,810	0.357	2.360	0.124	0.0112	0.119
7	2.94	1,880	0.362	2.390	0.236	0.0192	0.211
8	3.04	1,940	0.367	2.470	0.260	0.0220	0.250
9	3.13	2,010	0.367	2.555	0.302	0.0193	0.226
10	3.23	2,060	0.379	2.725	0.394	0.0232	0.280
11	3.33	2,140	0.383	2.755	0.548	0.0272	0.339
12	3.42	2,200	0.387	2.800	0.774	0.0242	0.310
13	3.52	2,270	0.392	2.930	0.862	0.0204	0.268
14	3.62	2,350	0.397	3.030	0.891	0.0112	0.152

Table 18Run No. 15

Mean water temperature = 83.0°F

Mean oil temperature = 84.0°F

Oil flow rate = 4.28 USGPM

 $N_{Re_o} = 417$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	2.29	1,490	0.306	2.480	0	0	0
2	2.54	1,620	0.316	2.520	0.011	0.0014	0.013
3	2.74	1,780	0.331	2.580	0.040	0.0036	0.037
4	2.94	1,920	0.338	2.660	0.163	0.0098	0.108
5	3.13	2,050	0.349	2.700	0.314	0.0190	0.222
6	3.33	2,175	0.365	2.775	0.553	0.0214	0.266
7	3.42	2,230	0.378	2.920	0.781	0.0178	0.228
8	3.52	2,300	0.369	2.960	0.760	0.0148	0.195
9	3.62	2,380	0.375	2.980	0.853	0.0136	0.184

Table 19Run No. 16

Mean water temperature = 83.5°F

Mean oil temperature = 80.5°F

Oil flow rate = 5.02 USGPM

 $N_{Re_o} = 465$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	2.44	1,605	0.298	3.690	0	0	0
2	2.64	1,735	0.309	3.730	0	0	0
3	2.64	1,750	0.310	3.910	0.042	0.0041	0.041
4	2.84	1,868	0.322	3.770	0.061	0.0045	0.048
5	3.04	2,000	0.330	3.955	0.202	0.0132	0.150
6	3.23	2,130	0.355	4.140	0.487	0.0188	0.227
7	3.42	2,255	0.362	4.425	0.846	0.0106	0.128
8	3.62	2,385	0.367	4.690	0.964	0.0052	0.0704
9	3.82	2,415	0.378	4.810	1	0	0

Table 20Run No. 17

Mean water temperature = 83.5°F

Mean oil temperature = 80.0°F

Oil flow rate = 6.93 USGPM

 $N_{Re_o} = 636$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	2.54	1,670	0.257	4.630	0	0	0
2	2.74	1,810	0.274	4.680	0.015	0.0012	0.012
3	2.74	1,810	0.272	4.600	0.101	0.0094	0.096
4	2.84	1,880	0.273	4.610	0.206	0.0158	0.168
5	2.94	1,930	0.276	4.780	0.384	0.0252	0.277
6	3.04	2,015	0.289	5.070	0.798	0.0236	0.268
7	3.23	2,140	0.308	5.345	0.978	0.0039	0.047
8	3.42	2,280	0.321	5.595	1	0	0

Table 21Run No. 18

Mean water temperature = 82.5°F

Mean oil temperature = 80.0°F

Oil flow rate = 8.13 USGPM

 $N_{Re_o} = 745$

No.	Water Flow Rate (USGPM)	N_{Re_w}	Water Depth (inches)	$\Delta P/\Delta L$ (cm of fluid)	γ	$\bar{n}$ ($\frac{\text{changes}}{\text{second}}$)	$\frac{\bar{n}d}{\bar{u}}$
1	2.74	1,780	0.253	5.030	0.278	0.0192	0.197
2	2.84	1,850	0.273	5.285	0.507	0.0307	0.326
3	2.94	1,920	0.279	5.660	0.796	0.0272	0.299
4	3.04	1,980	0.283	5.870	0.966	0.0050	0.057
5	3.13	2,044	0.285	6.035	0.999	0.0010	0.012
6	3.23	2,120	0.290	6.070	1	0	0
7	3.42	2,240	0.301	6.195	1	0	0

Table 22Laminar-Turbulent Transition Water Reynolds Numbers

N_{Re_o}	N_{Re_w} ($\gamma=0.2$)	N_{Re_w} ($\gamma=0.5$)	N_{Re_w} ($\gamma=0.8$)	Interface Position (Inches from Bottom, $\gamma=0.5$)
0	2,570	2,680	2,790	1.007
26	2,680	2,800	2,890	0.770
38		2,670		
58	2,380	2,450	2,560	0.637
61	2,430	2,500	2,570	0.640
76	2,430	2,520	2,590	0.634
105	2,380	2,470	2,610	0.595
174	2,160	2,250	2,390	0.503
234	1,820	1,990	2,110	0.440
293	1,650	1,760	1,920	0.386
320	1,650	1,760	2,170	0.375
353	1,850	2,120	2,240	0.370
417	1,960	2,160	2,290	0.365
465	2,000	2,130	2,230	0.355
636	1,880	1,950	2,020	0.280
745	1,740	1,850	1,920	0.273

Table 23

Intermittent Flow for Different Probe Positions

Water Flow Rate = 3.82 USGPM $N_{Re_w} = 2420$

Oil Flow Rate = 0.724 USGPM $N_{Re_o} = 65$

No.	Water Temp., °F	Oil Temp., °F	Probe Position (inches from bottom)	γ	$\bar{n}$ (changes/ second)
1	80.2	79.8	0.180	0.154	0.0272
2	80.4	79.0	0.560	0.172	0.0284
	80.5	78.5	0.560		
3	80.5	78.5	0.180	0.225	0.0274
4	80.5	79.0	0.370	0.248	0.0348
	81.0	79.5	0.370		
5	81.0	79.0	0.180	0.260	0.0362
Average No. 1 and No. 3				0.189	0.0273
Average No. 3 and No. 5				0.242	0.0318
Average No. 1, No. 3 and No. 5				0.213	0.0303
Average No. 2 and No. 4				0.210	0.0316

Water Flow Rate = 3.23 USGPM $N_{Re_w} = 2050$

Oil Flow Rate = 5.02 USGPM $N_{Re_o} = 450$

1	80.8	78.5	0.180	0.304	0.0224
2	80.8	78.5	0.275	0.300	0.0188
	80.8	78.5	0.275		
3	80.1	78.5	0.180	0.288	0.0175
Average No. 1 and No. 3				0.296	0.0199

Water Flow Rate = 2.96 USGPM $N_{Re_w} = 1865$

Oil Flow Rate = 7.08 USGPM $N_{Re_o} = 632$

1	80.0	78.8	0.180	0.103	0.0087
2	80.0	78.5	0.225	0.119	0.0111
	80.4	78.5	0.225		
3	80.4	78.0	0.180	0.121	0.0216
Average No. 1 and No. 3				0.112	0.0097

Water Flow Rate = 3.13 USGPM $N_{Re_w} = 1985$

Oil Flow Rate = 7.08 USGPM $N_{Re_o} = 632$

1	80.5	78.0	0.180	0.536	0.0225
2	80.5	78.0	0.230	0.600	0.0249
	80.7	78.0	0.230		
3	80.7	78.0	0.180	0.660	0.0288
Average No. 1 and No. 3				0.598	0.0257

Table 24

Average Time of Laminar-Turbulent Flow Cycle

N_{Re_w}	$\frac{\bar{n}d}{\bar{u}}$	γ	Dimensionless Time of Lam.- Turb. Cycle $\frac{\bar{t}_u}{\bar{d}}$	Dimensionless Time of Turb. Spot $\frac{\bar{t}_T}{\bar{d}}$
$N_{Re_o} = 0$ (Data from Figures 19 and 22)				
2,400	0.03	0.01	33.30	0.333
2,450	0.08	0.04	12.50	0.500
2,500	0.155	0.095	6.45	0.612
2,600	0.360	0.262	2.78	0.729
2,650	0.507	0.400	1.97	0.788
2,700	0.507	0.565	1.97	1.112
2,800	0.340	0.815	2.94	2.395
2,900	0.208	0.931	5.19	4.830
3,000	0.115	0.976	8.70	8.490
$N_{Re_o} = 58$ (Data from Figures 17 and 20)				
2,150	0.111	0.032	9.0	0.288
2,200	0.175	0.037	5.72	0.212
2,300	0.339	0.083	2.95	0.245
2,400	0.535	0.260	1.87	0.486
2,450	0.651	0.500	1.54	0.770
2,500	0.765	0.678	1.31	0.889
2,550	0.526	0.792	1.90	1.505
2,600	0.370	0.855	2.70	2.31
2,700	0.198	0.916	5.05	4.62
2,800	0.105	0.942	9.52	8.97
$N_{Re_o} = 636$ (Data from Figures 18 and 21)				
1,750	0.064	0.040	15.60	0.624
1,800	0.092	0.072	10.90	0.785
1,850	0.138	0.140	7.24	1.012
1,900	0.203	0.267	4.93	1.315
1,950	0.330	0.482	3.03	1.460
1,975	0.345	0.610	2.90	1.77
2,000	0.328	0.740	3.05	2.260
2,025	0.205	0.825	4.88	4.02
2,050	0.145	0.882	6.90	6.08
2,100	0.081	0.945	12.34	11.68

APPENDIX B
FIGURES

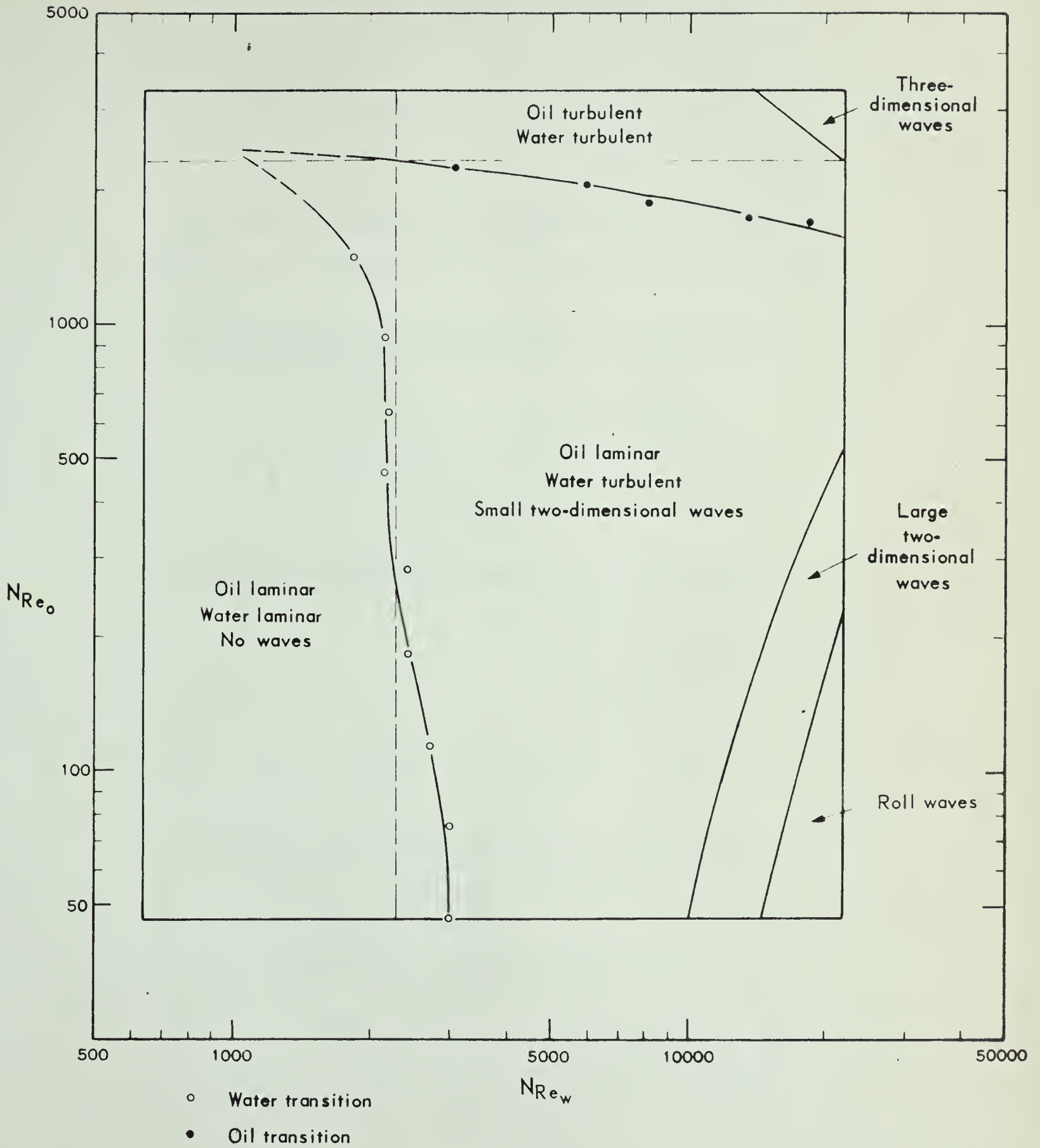


FIGURE 1. Flow regimes and interfacial structure.

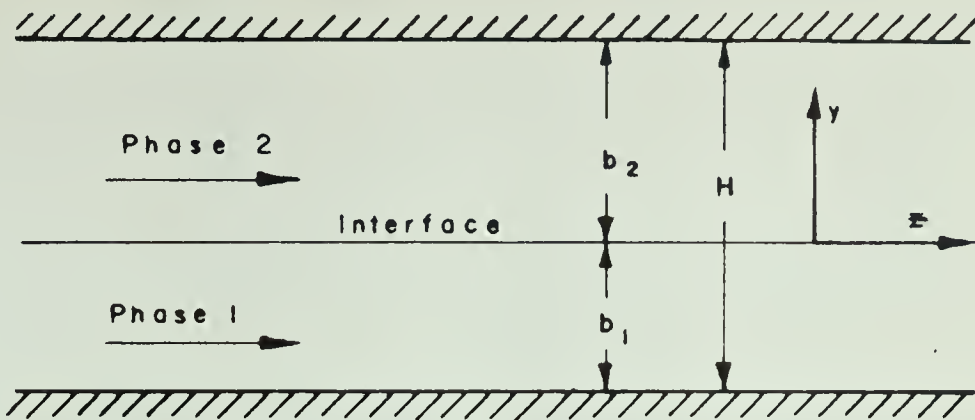


FIGURE 2a. Model of two-phase flow between parallel plates

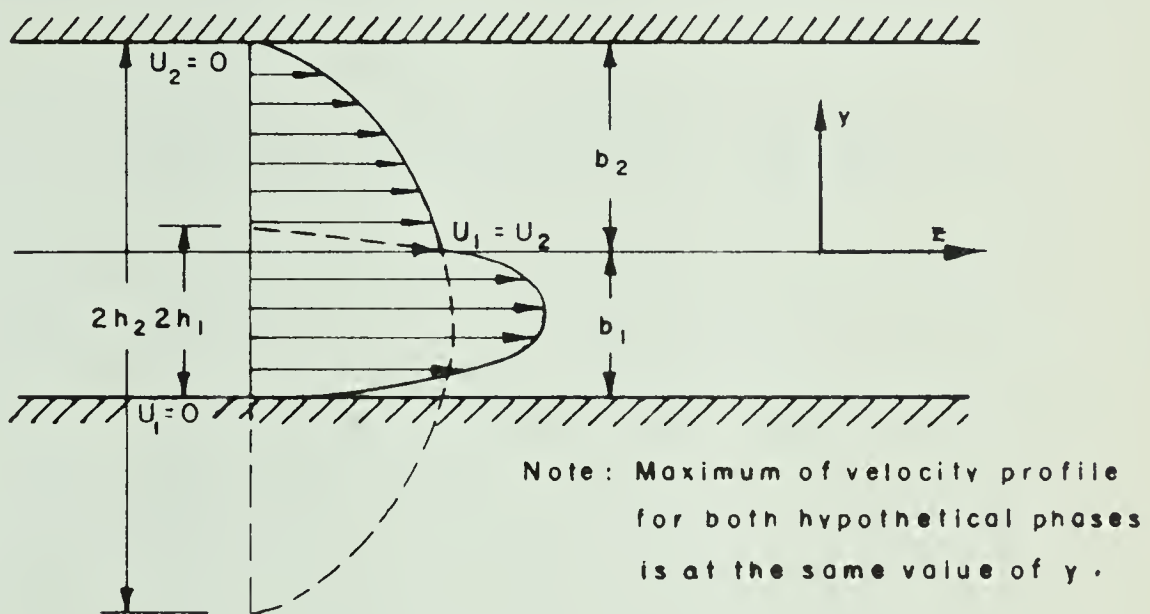


FIGURE 2b. Model of hypothetical parallel plate separation for two-phase flow

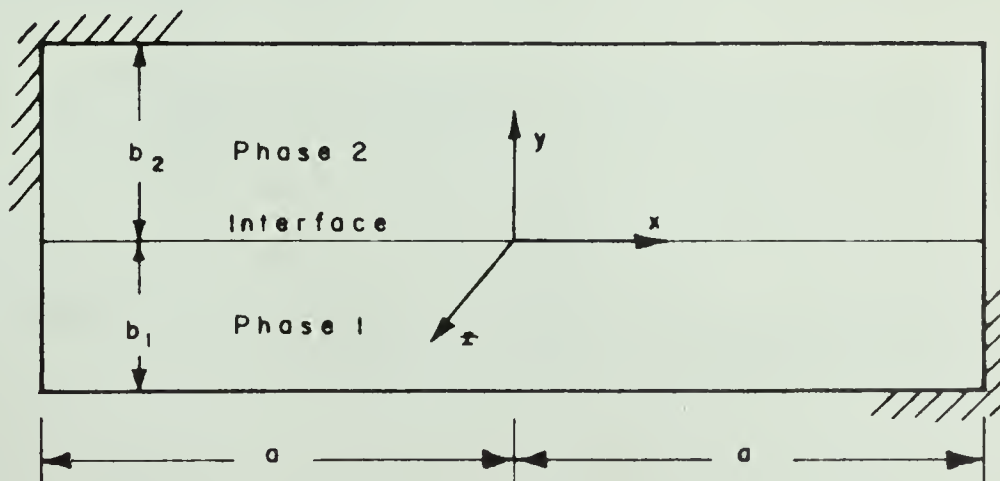


FIGURE 2c. Model of two-phase flow in rectangular conduit

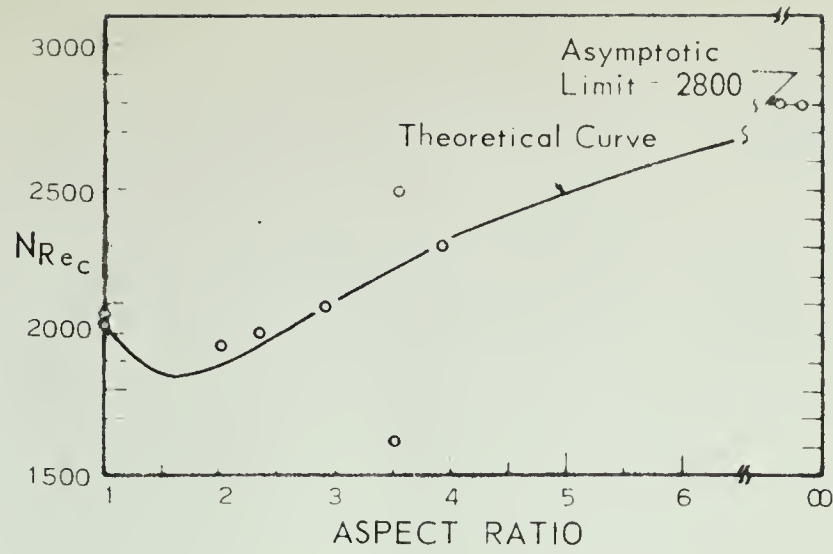


FIGURE 3. Theoretical critical Reynolds number as a function of aspect ratio (Reference 17)

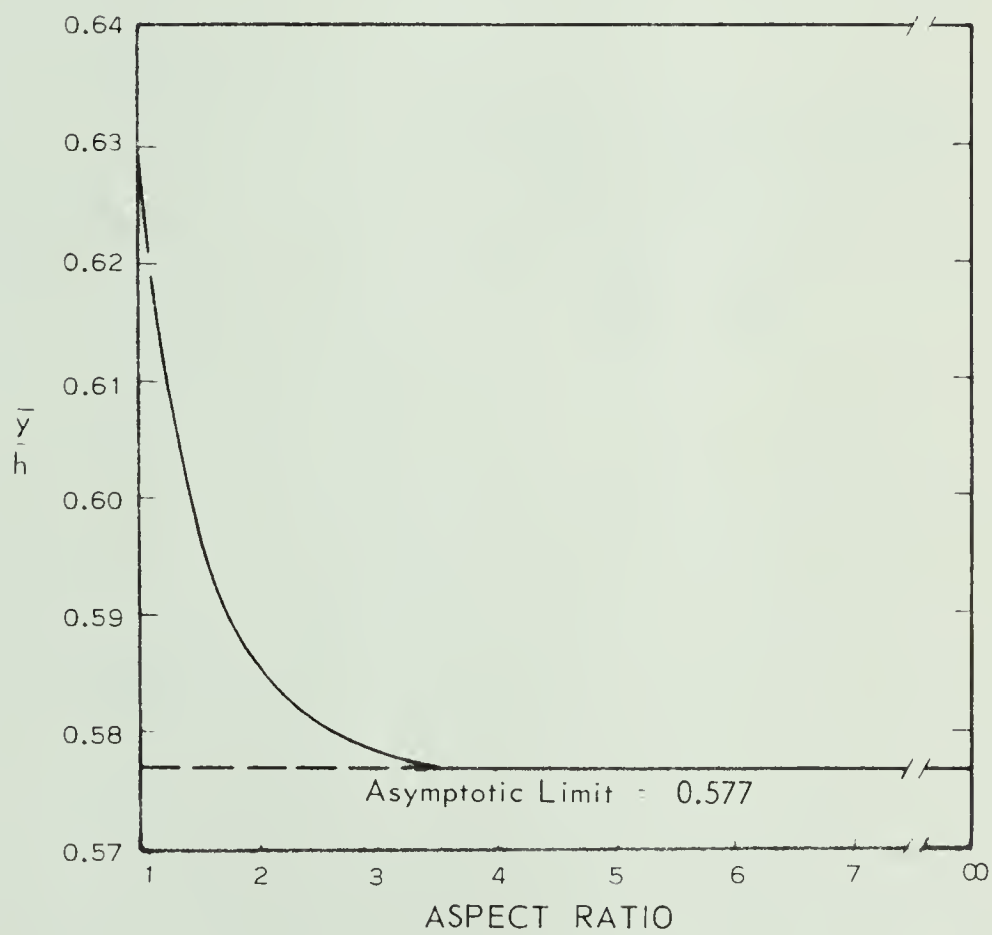


FIGURE 4. Spatial coordinates of theoretical location of turbulence inception as a function of duct aspect ratio (Reference 17)

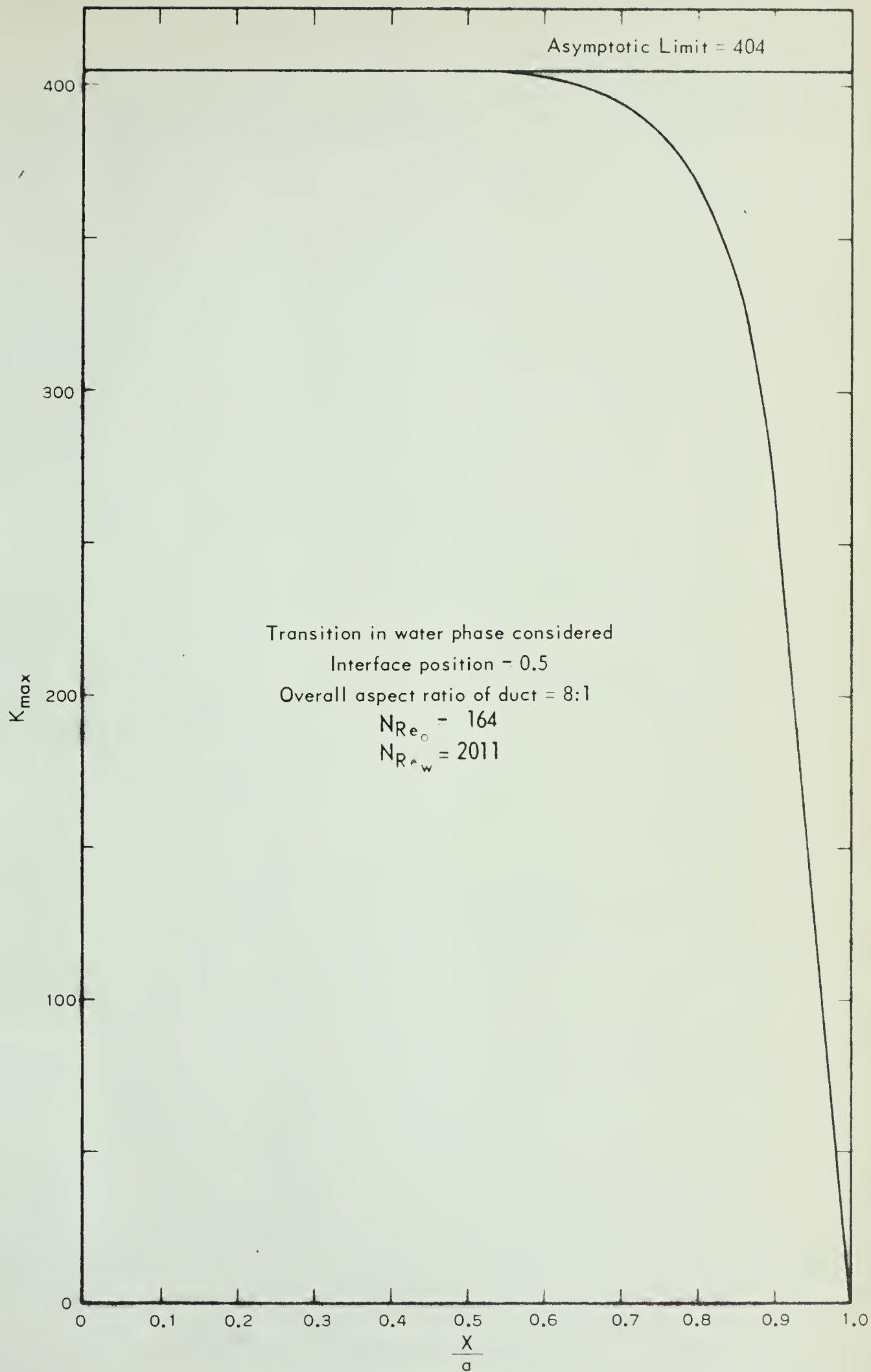


FIGURE 5. Stability parameter $K_{\max}$ as a function of x/a for two phase oil-water flow

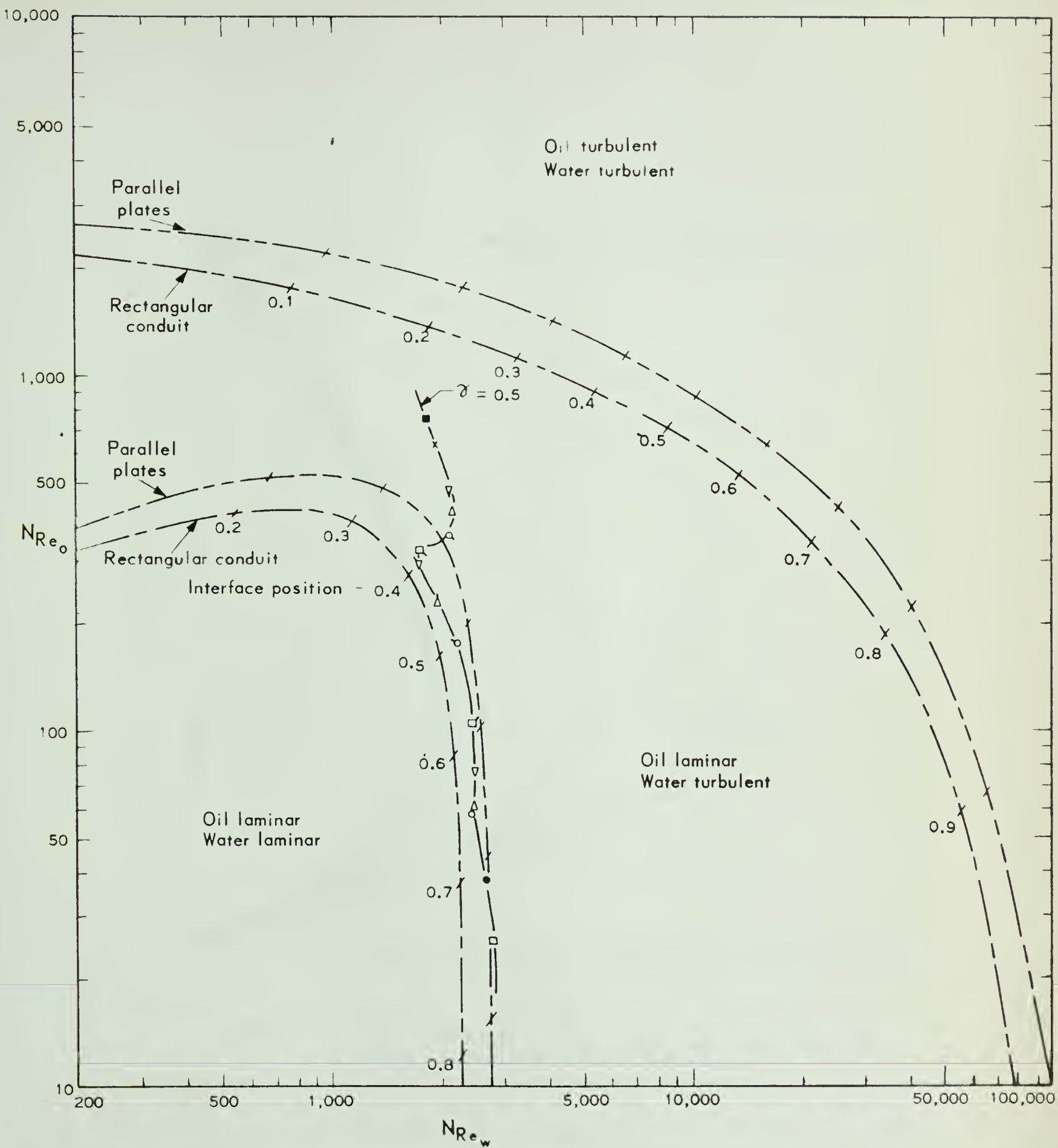


FIGURE 6. Theoretical flow regimes

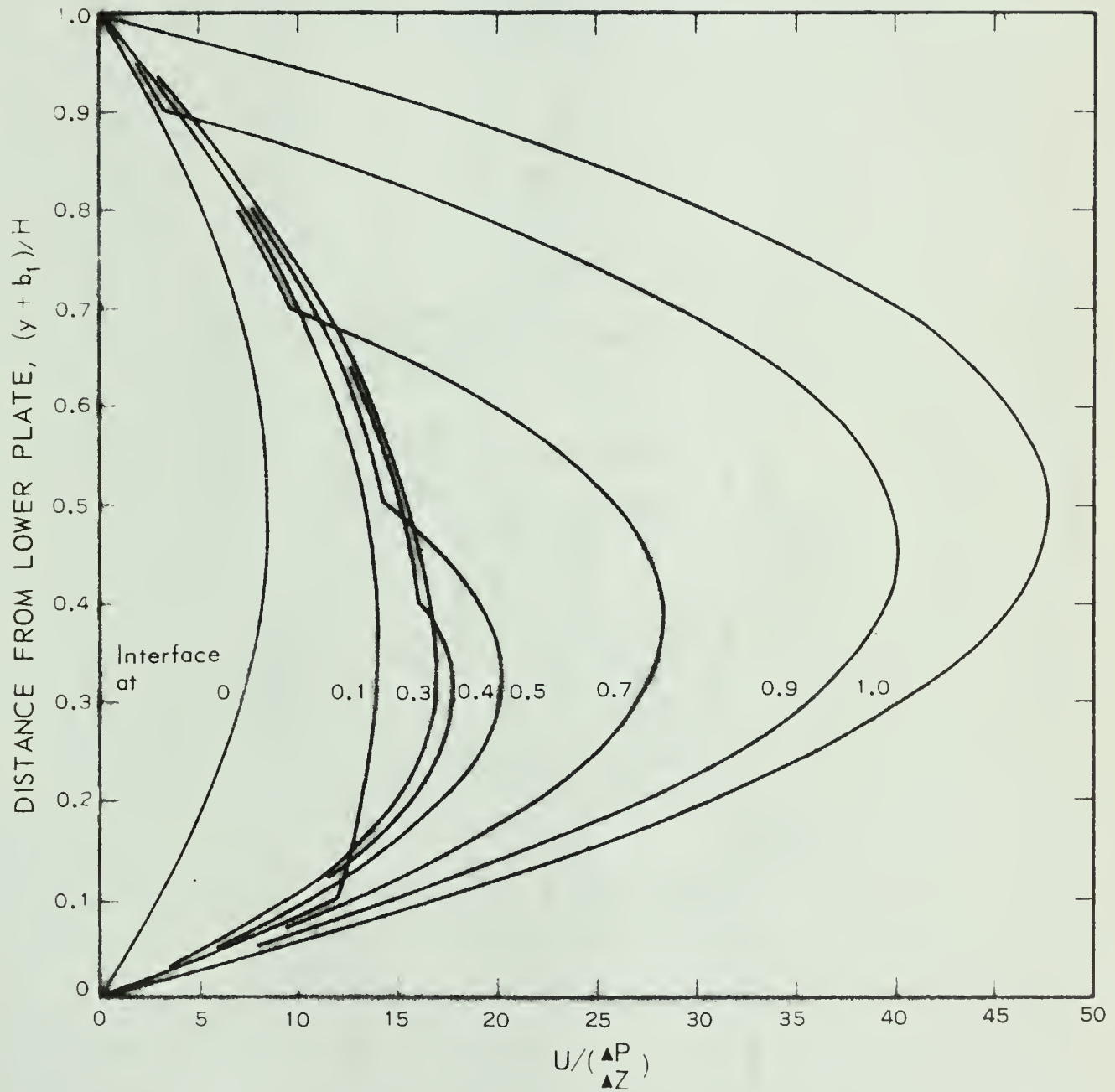


FIGURE 7. Laminar velocity distribution at central vertical cross-section of a rectangular conduit for an oil-water system

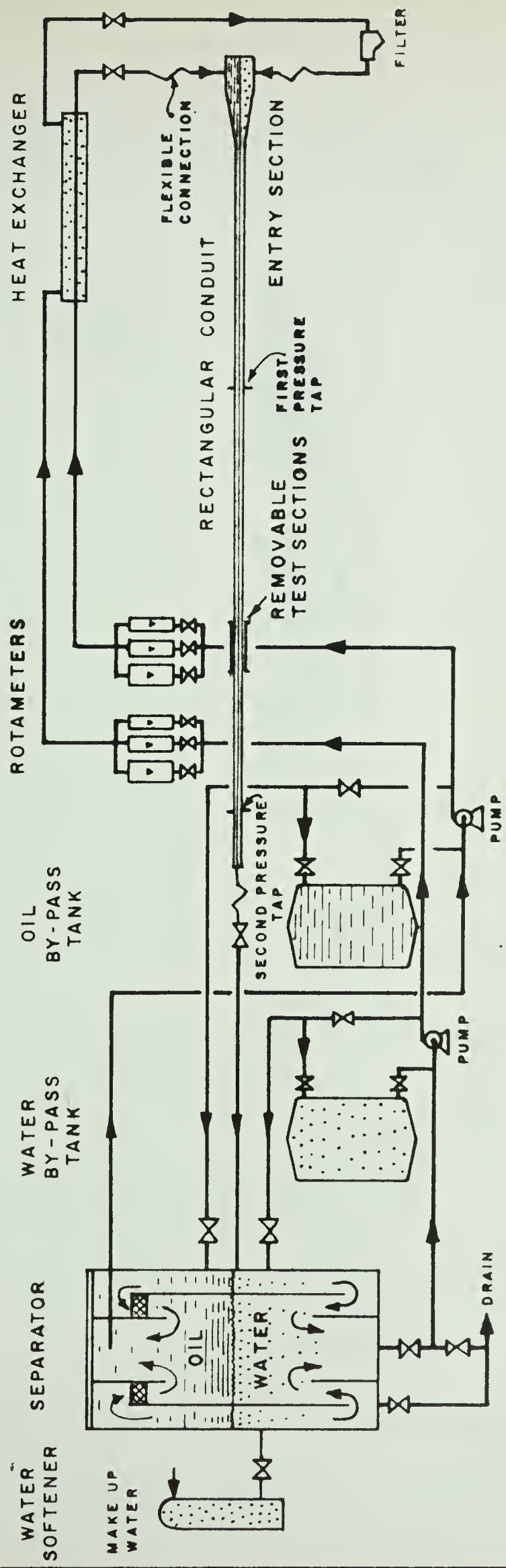


FIGURE 8. Flow system equipment

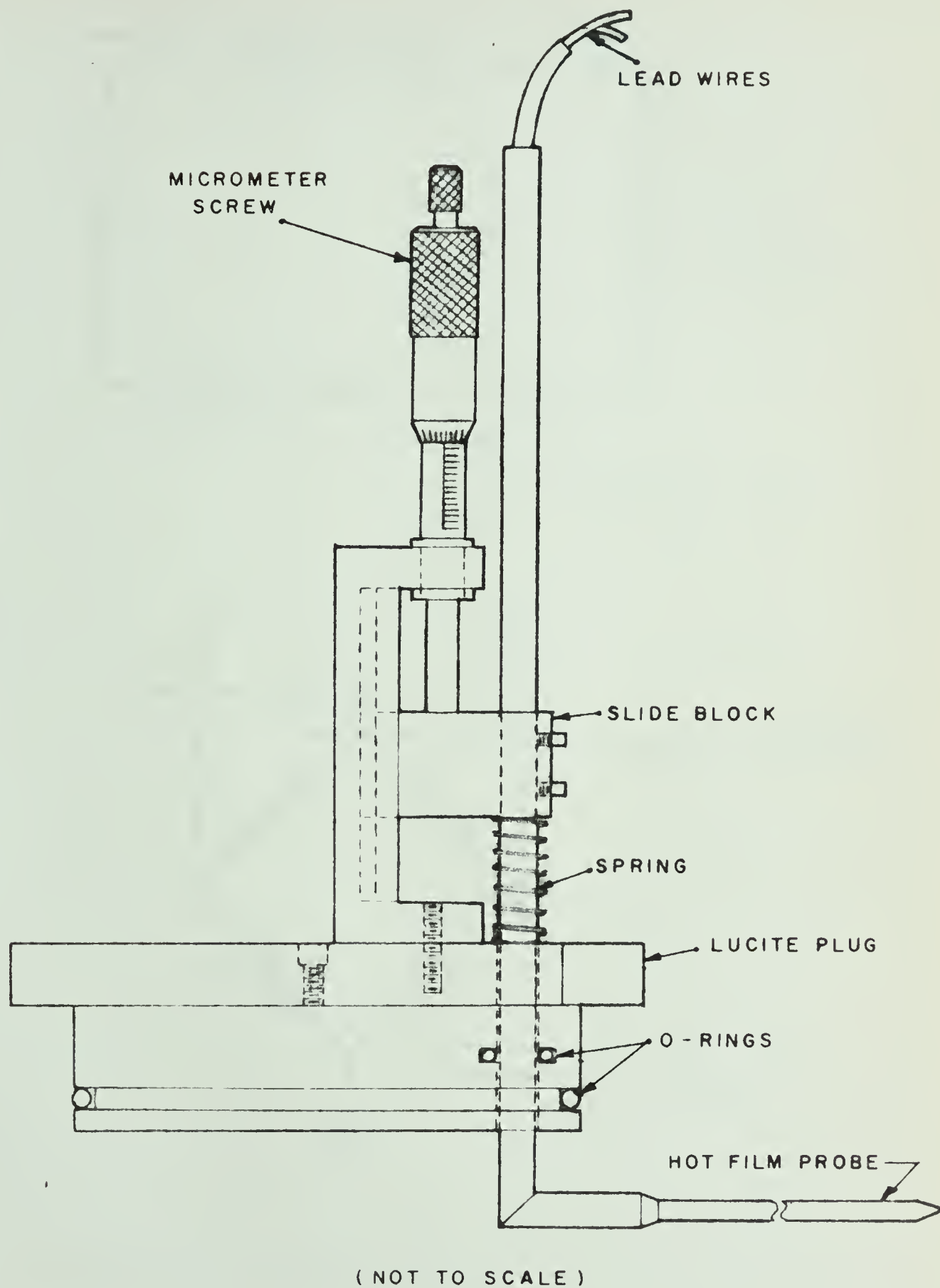


FIGURE 9. Hot - film probe traversing mechanism

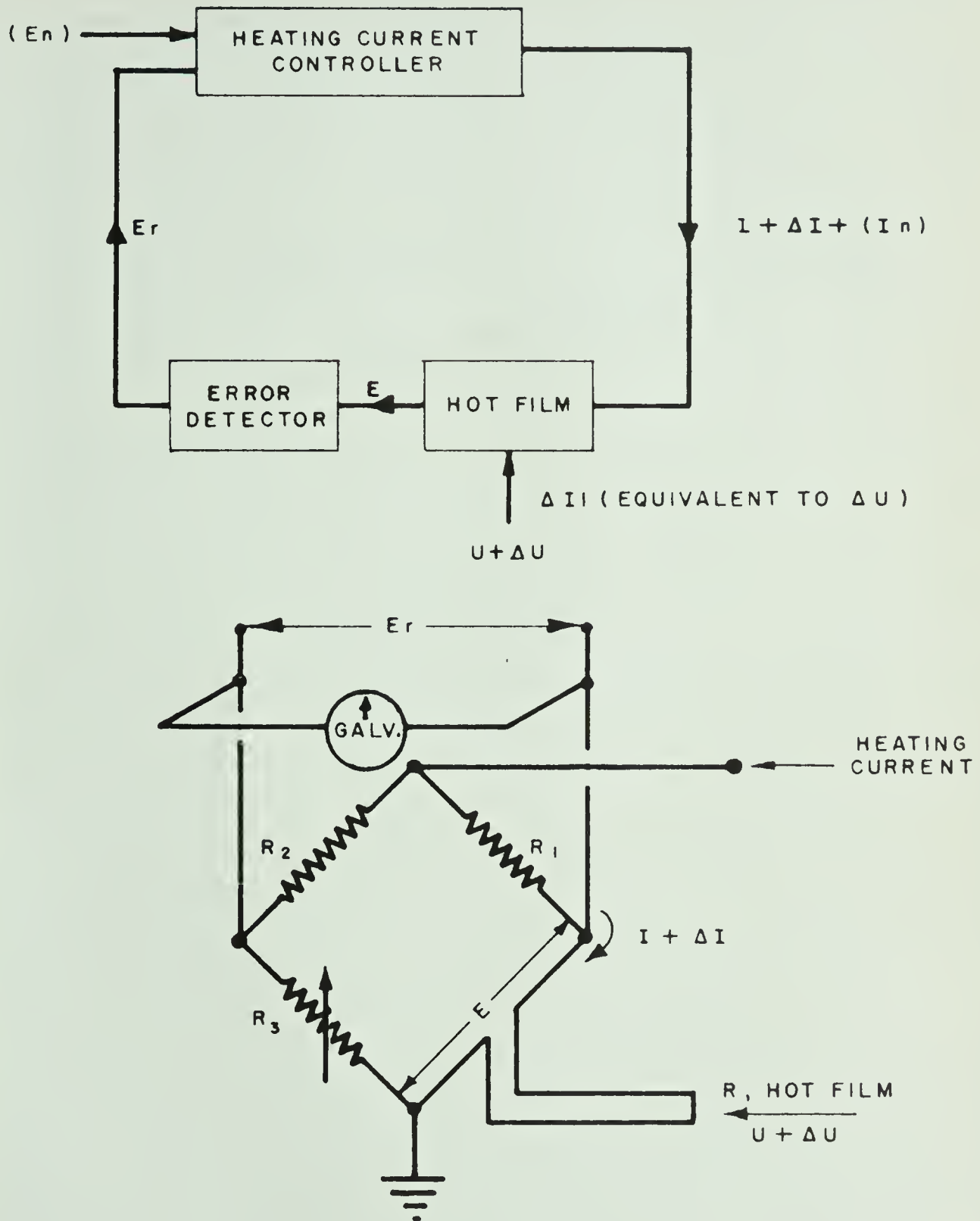


FIGURE 10 Constant-temperature control circuit.

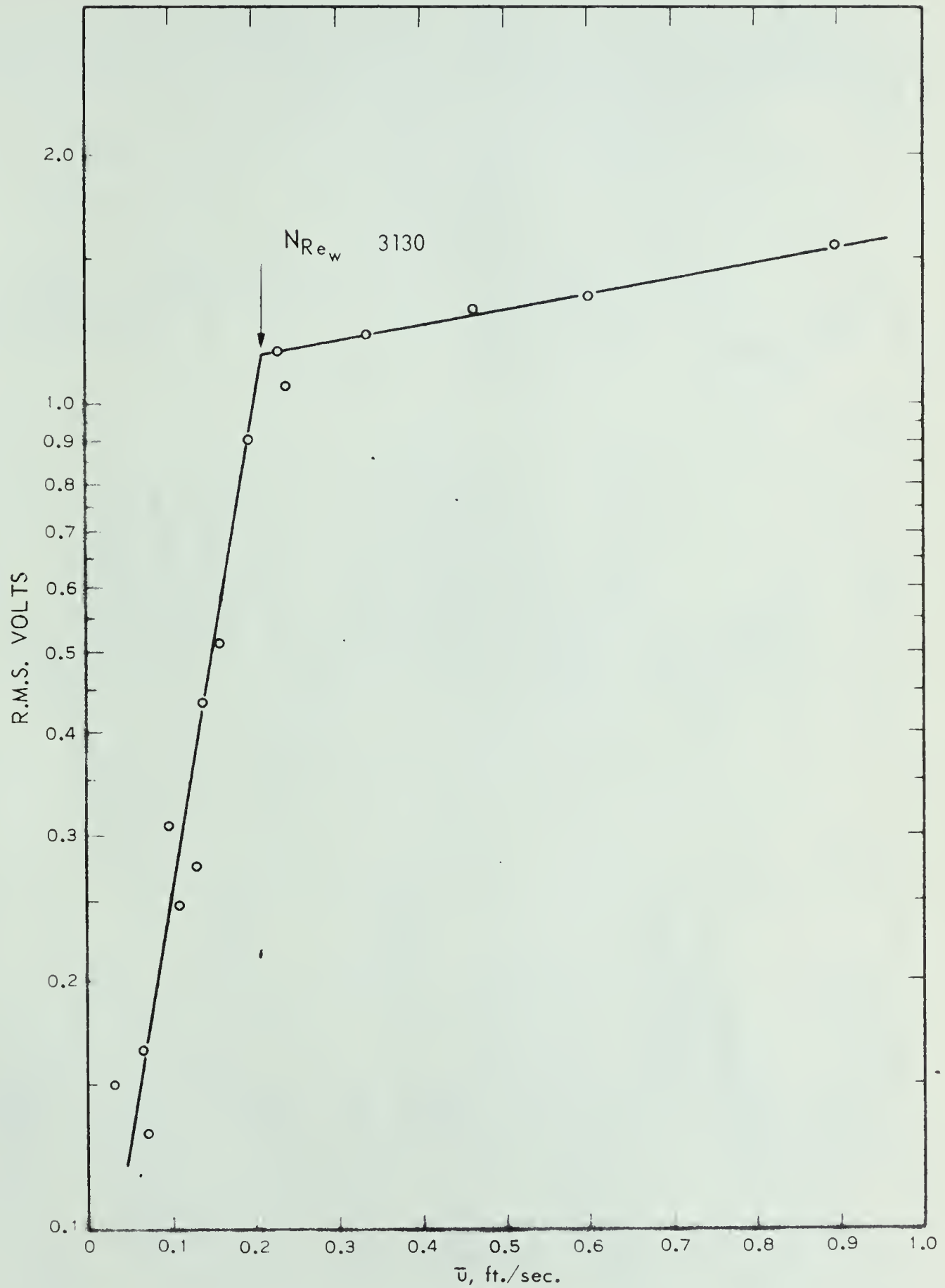


FIGURE 11. Root-mean-square voltage as a function of average velocity

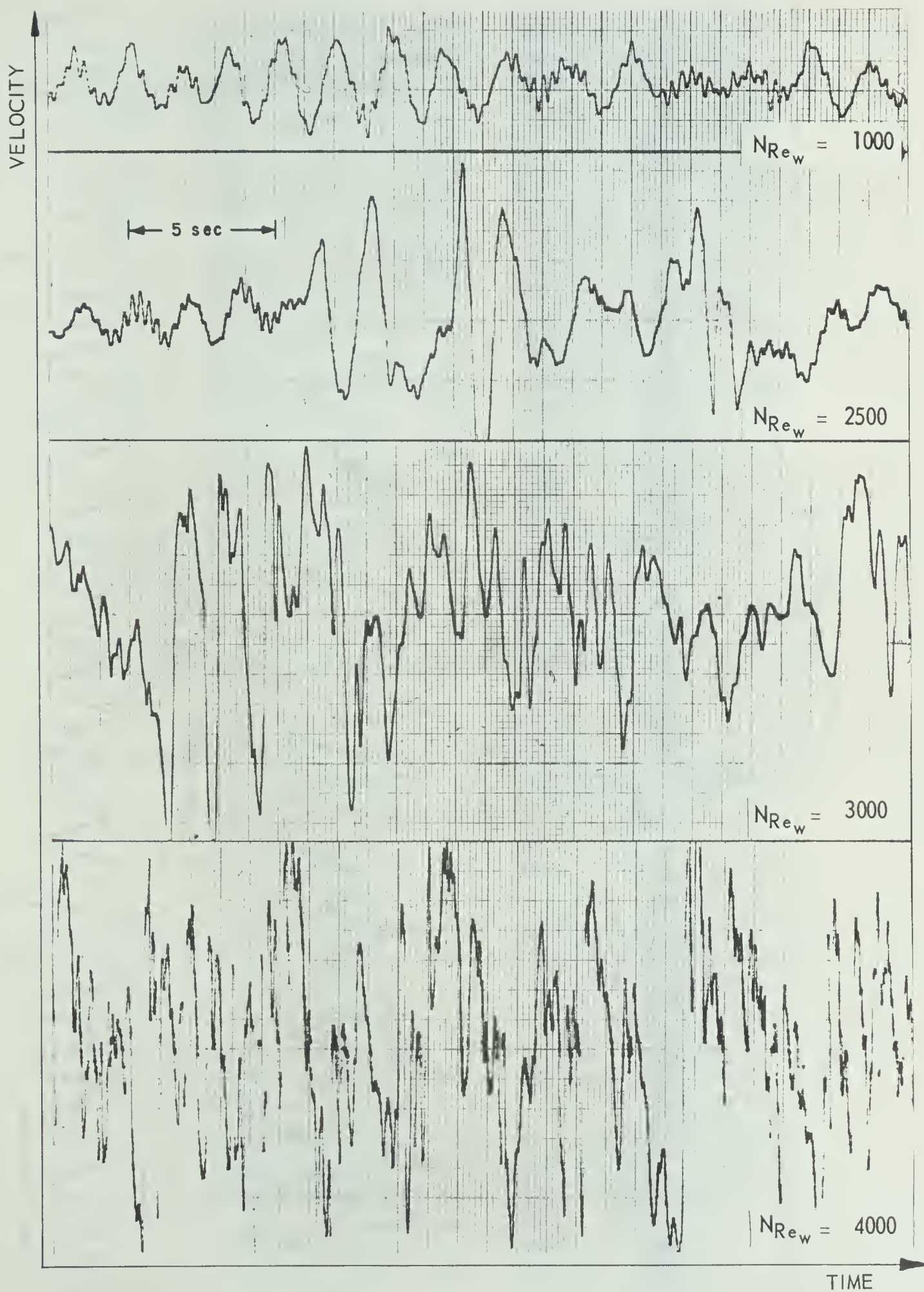


FIGURE 12. Velocity fluctuation frequencies for water flowing in a rectangular conduit

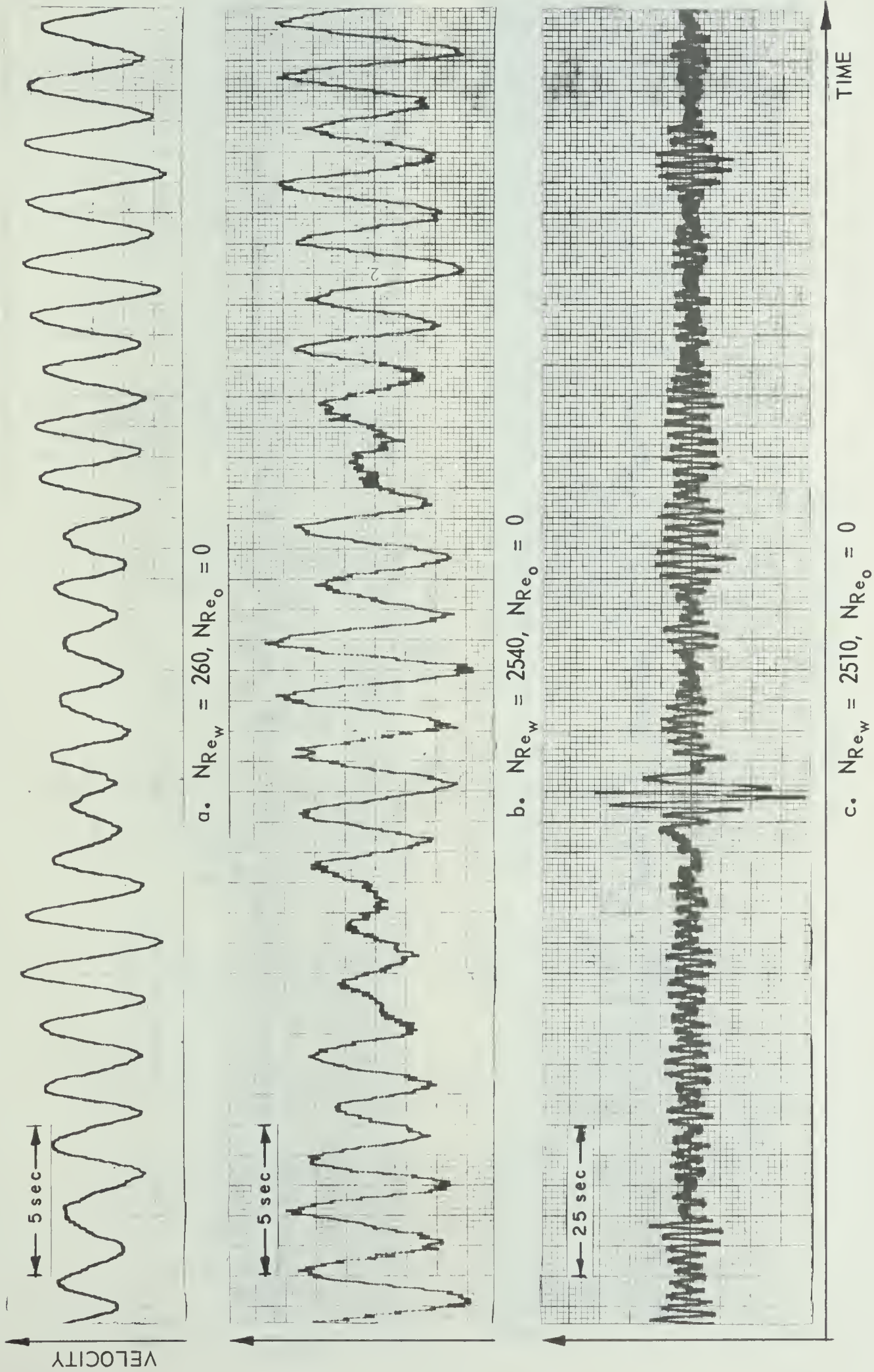
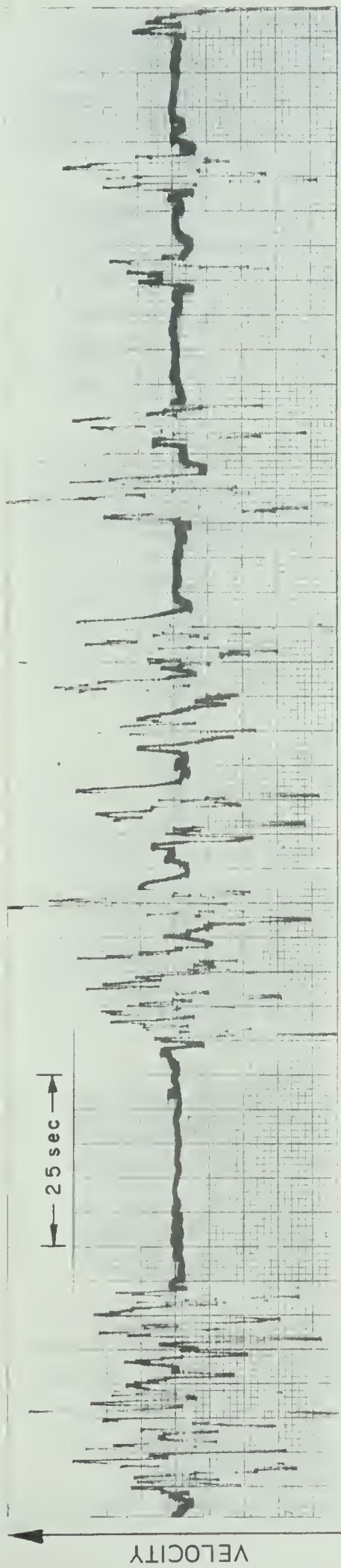
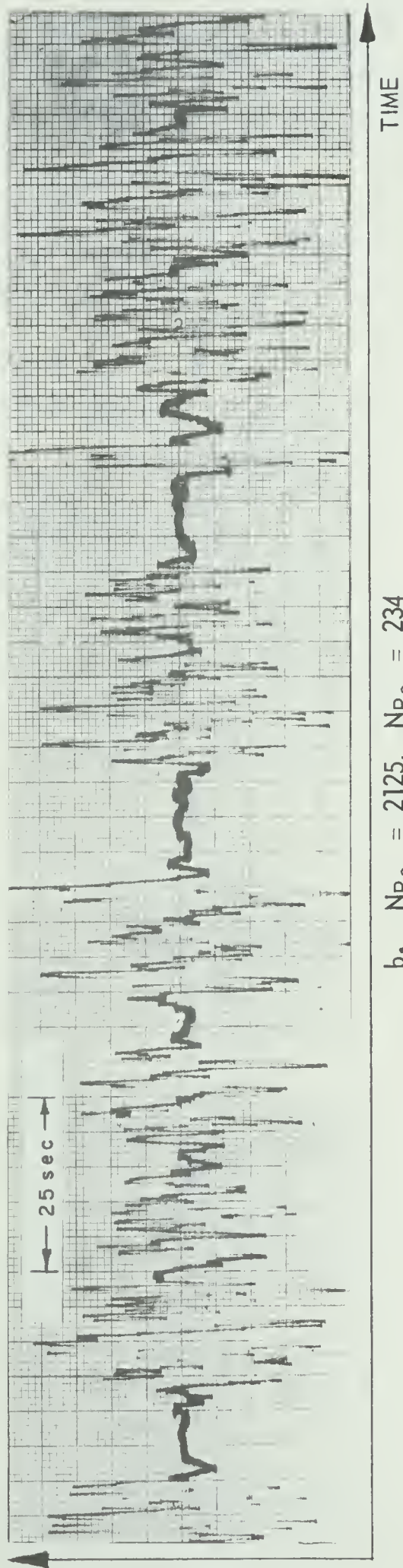


FIGURE 13. Sinusoidal velocity fluctuations



a. $N_{Re_w} = 2000$, $N_{Re_o} = 234$



b. $N_{Re_w} = 2125$, $N_{Re_o} = 234$

FIGURE 14. Typical hot-film anemometer output records for $N_{Re_o} < 330$

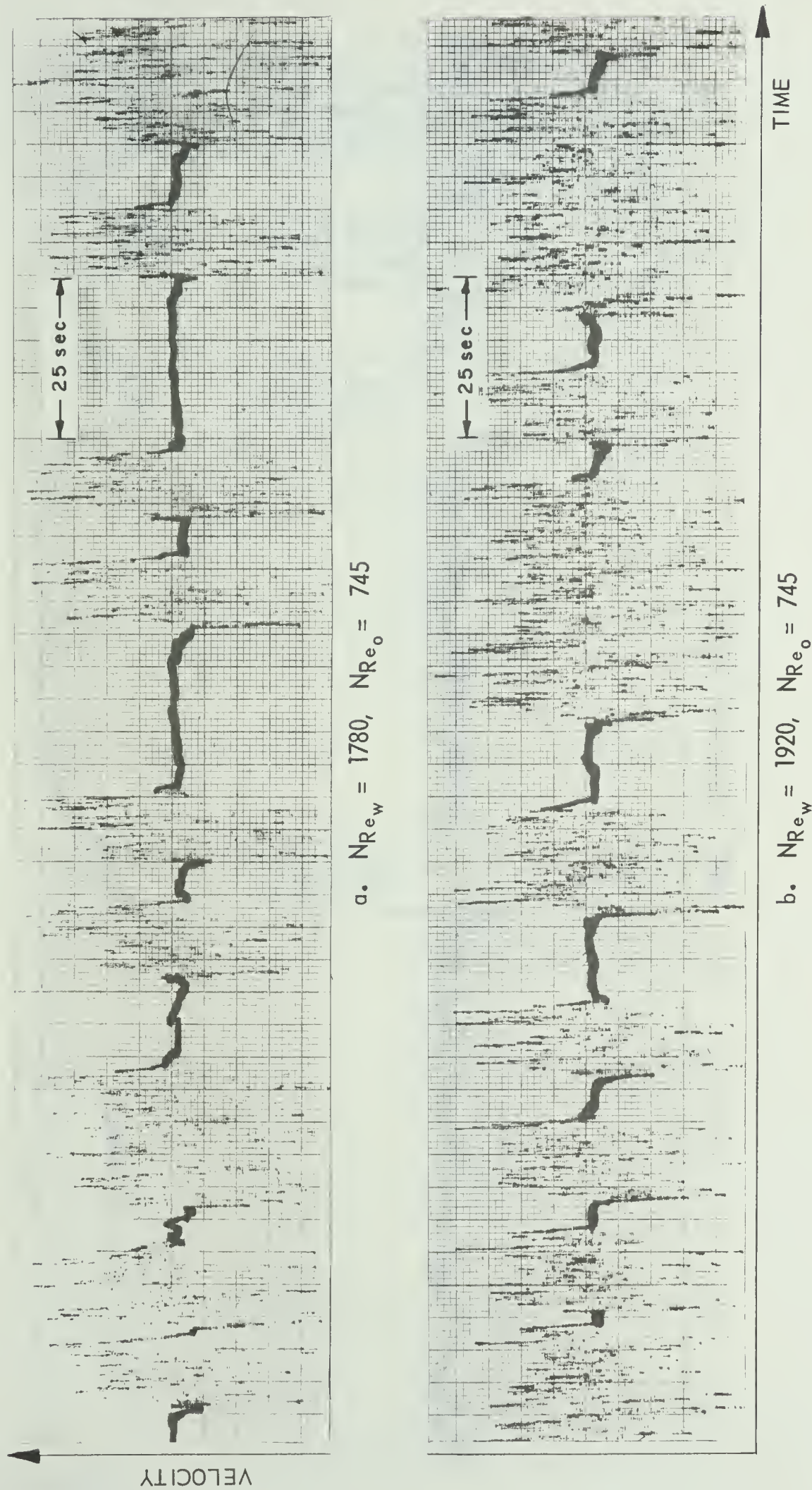


FIGURE 15. Typical hot-film anemometer output records for $N_{Re_o} > 330$

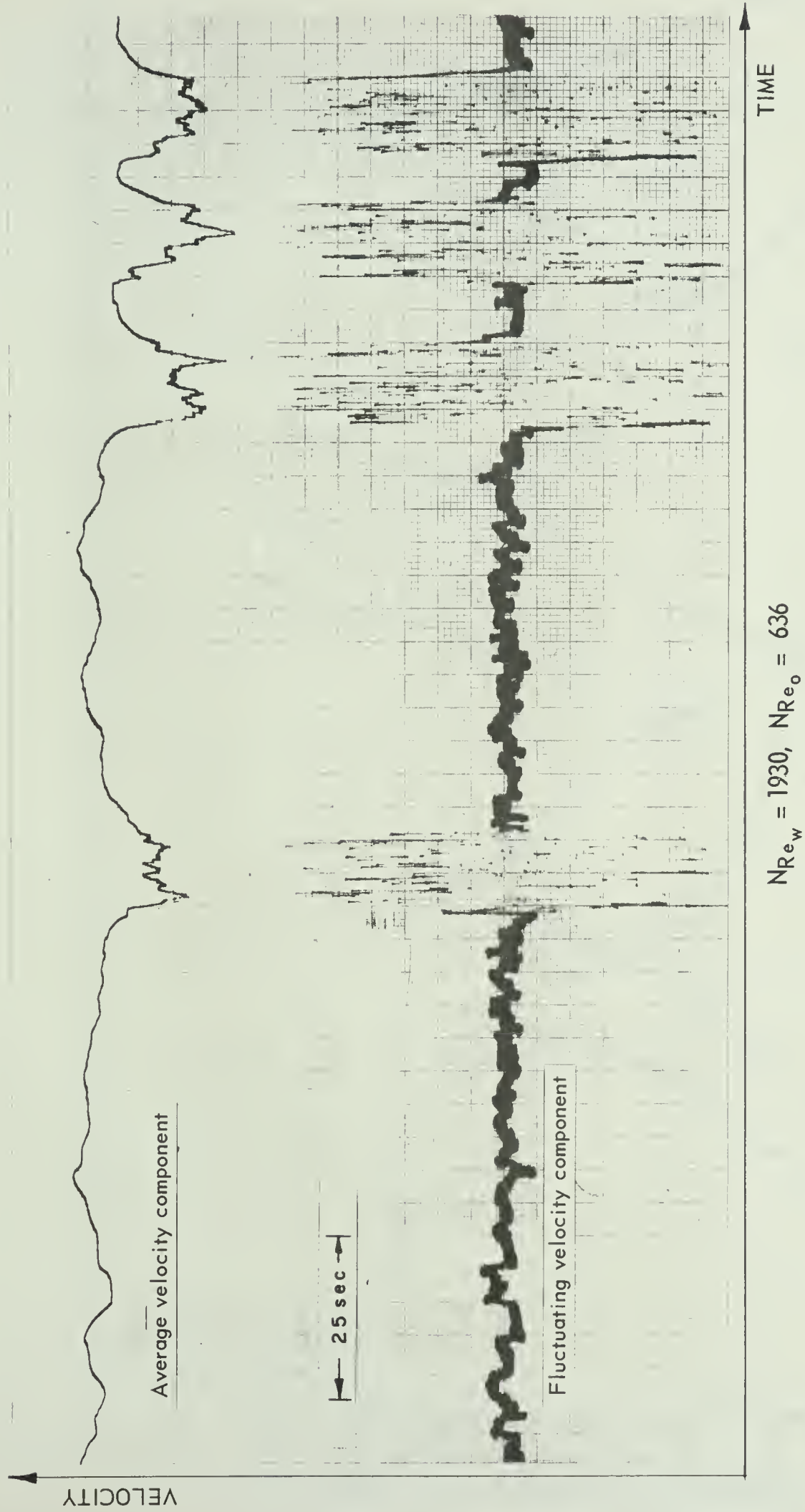


FIGURE 16. Closely spaced, constant length turbulent spots

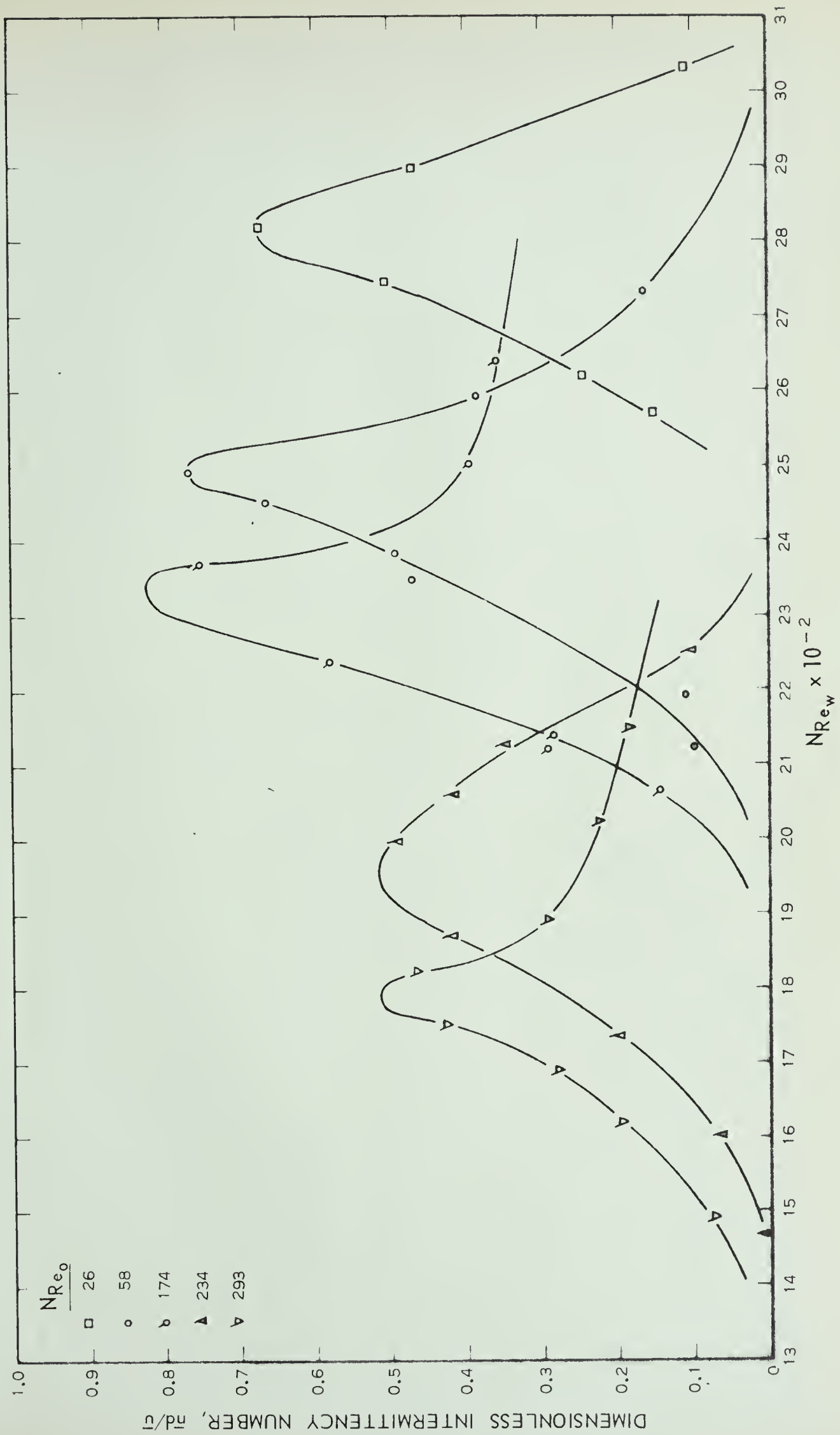


FIGURE 17. Intermittency number as a function of water Reynolds number
Oil Reynolds numbers 26 to 320

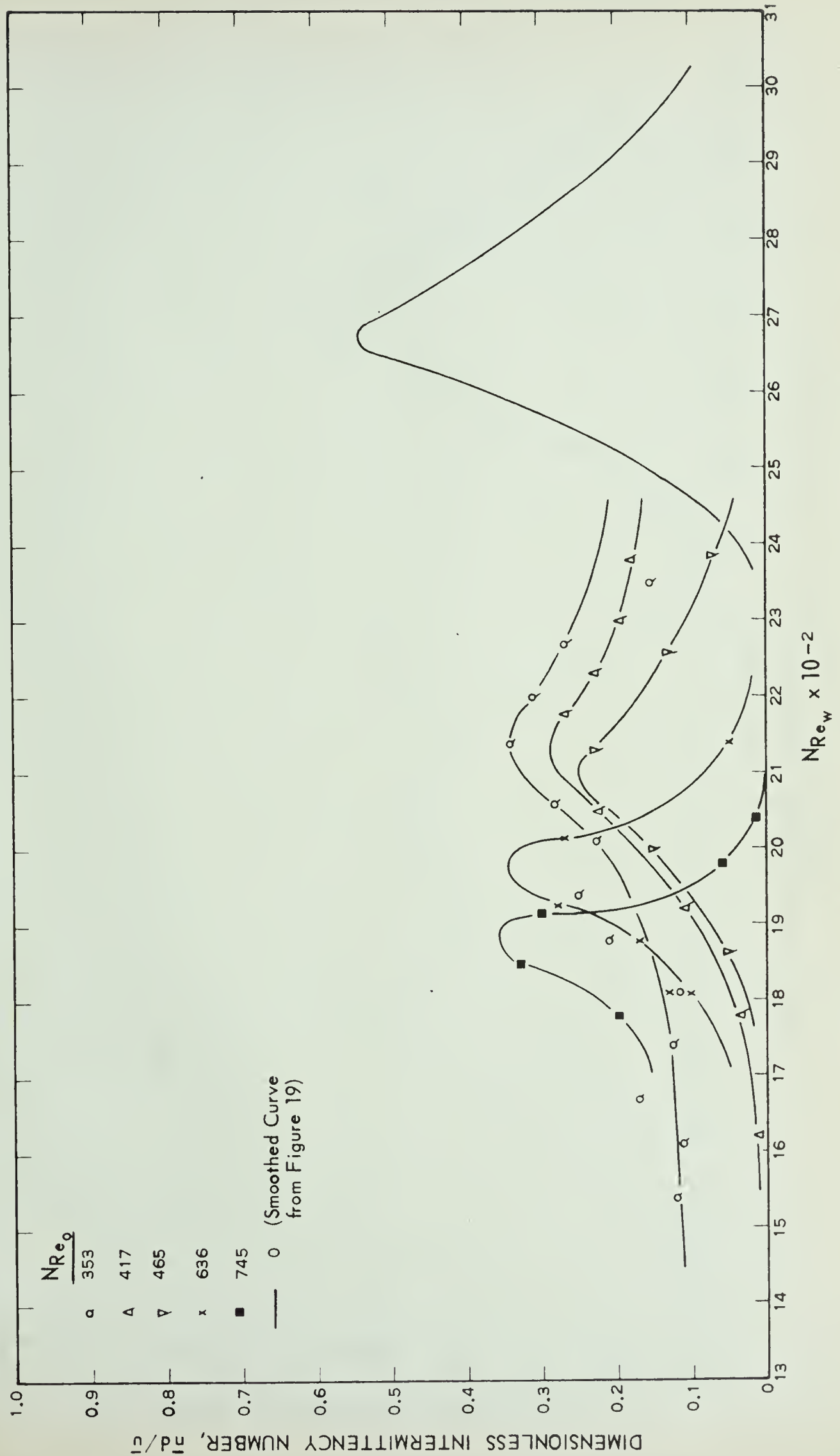


FIGURE 18. Intermittency number as a function of water Reynolds number
Oil Reynolds numbers 353 to 745

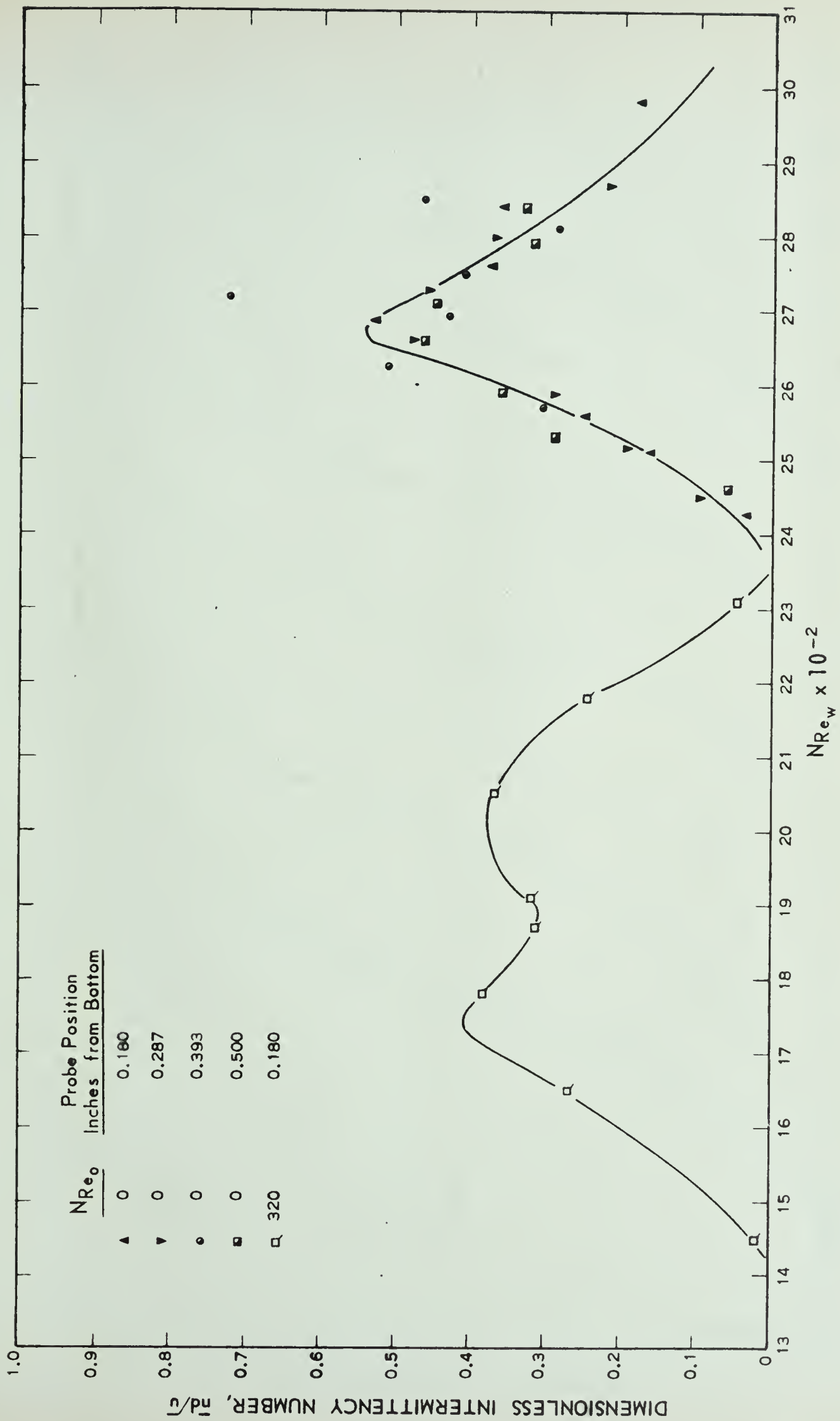


FIGURE 19. Intermittency number as a function of water Reynolds number
Oil Reynolds numbers 0 and 320

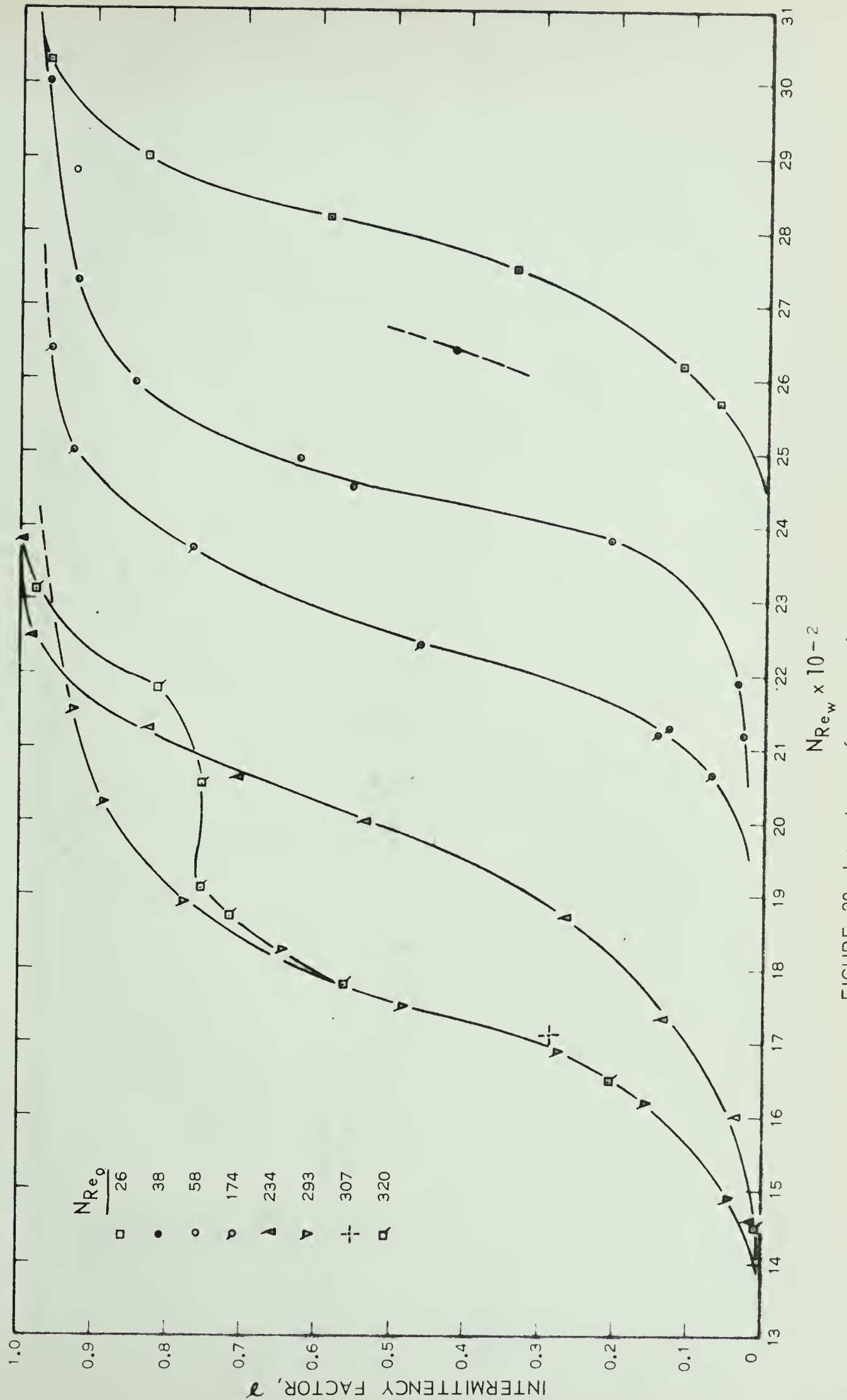


FIGURE 20. Intermittency factor as a function of water Reynolds number
Oil Reynolds numbers 26 to 320

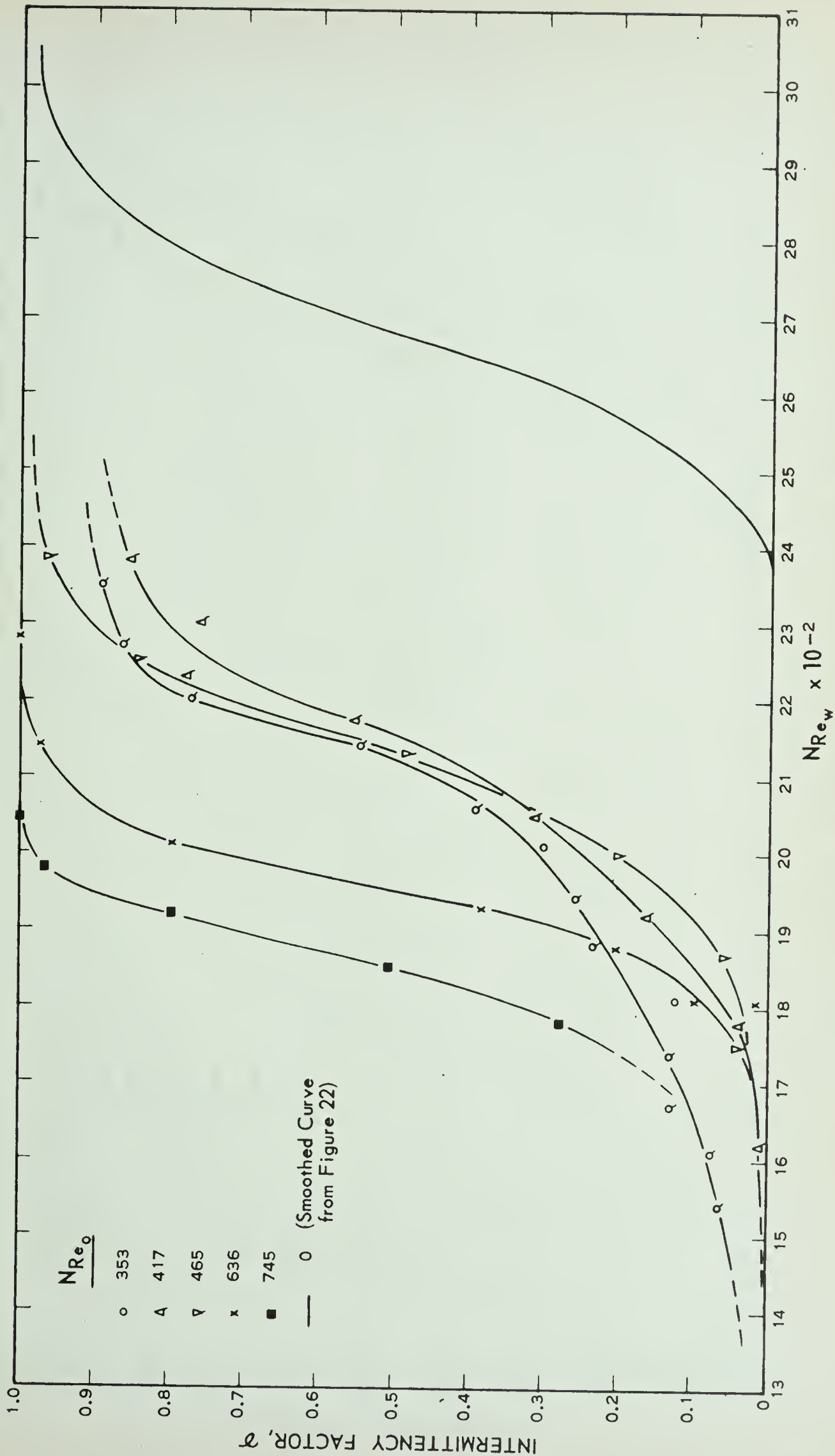


FIGURE 21. Intermittency factor as a function of water Reynolds number
Oil Reynolds numbers 353 to 745

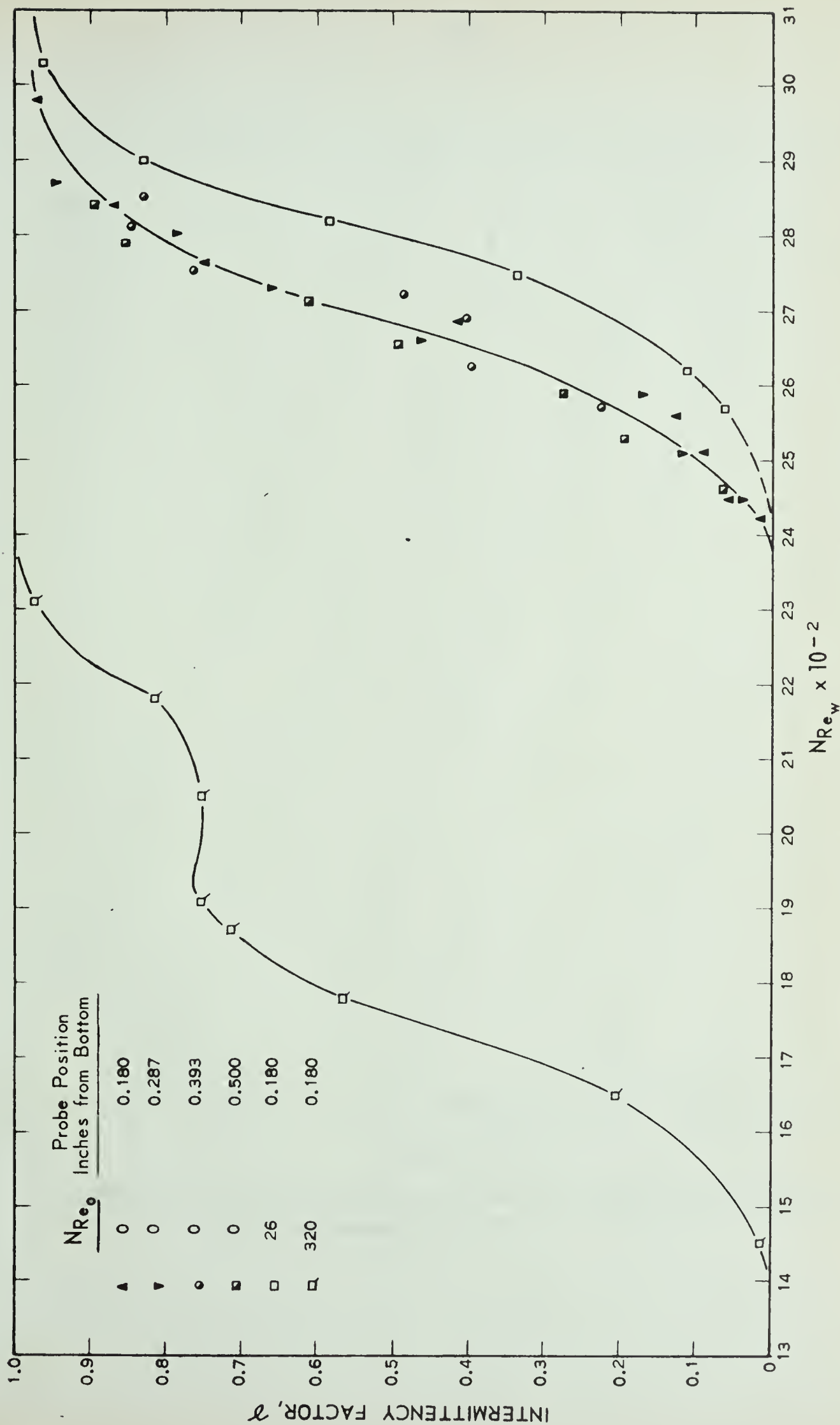
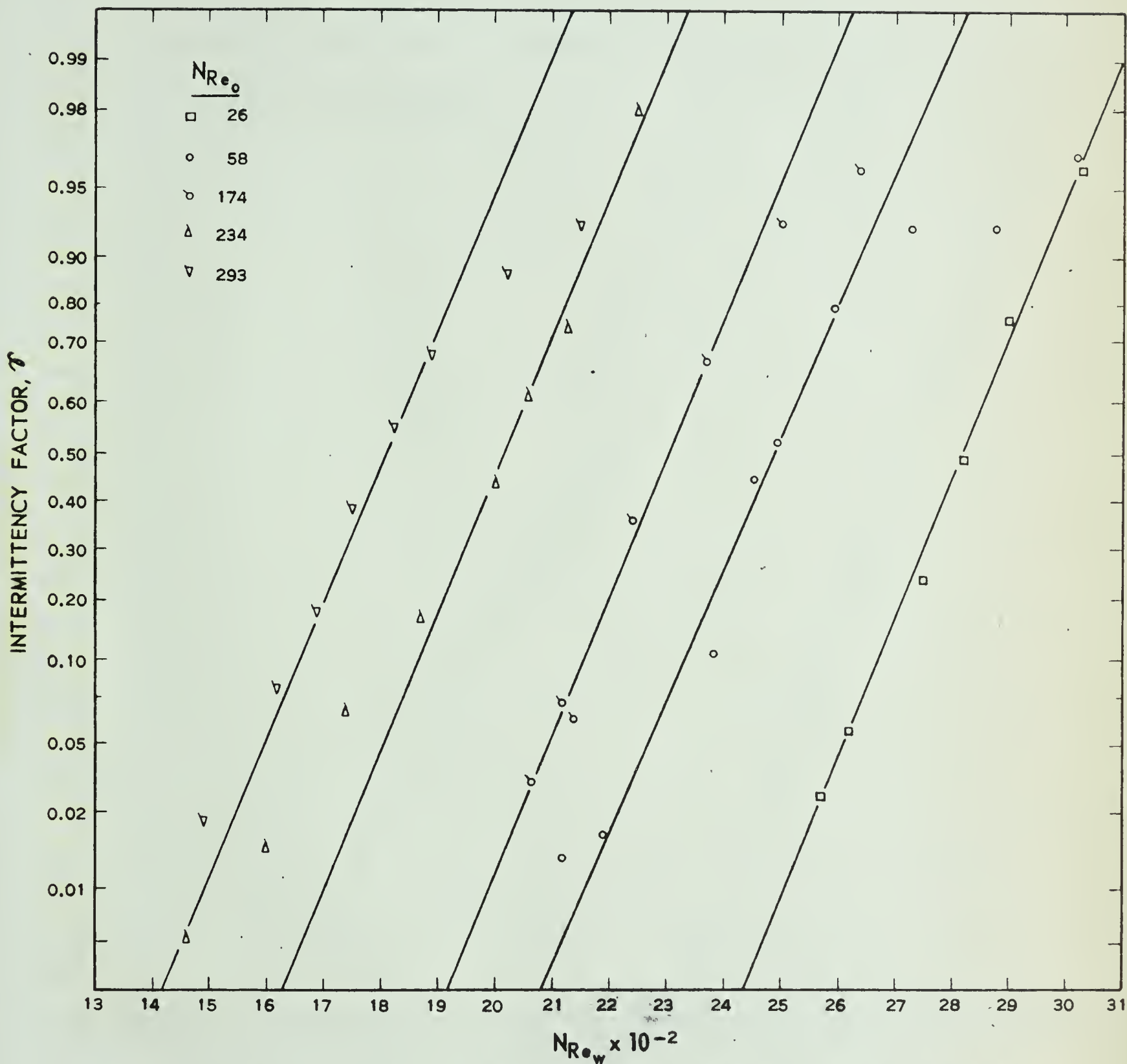


FIGURE 22. Intermittency factor as a function of water Reynolds number
Oil Reynolds numbers 0, 26 and 320



NUMBER 23. Probability plot of intermittency factor versus water Reynolds number
Oil Reynolds numbers 26 to 293

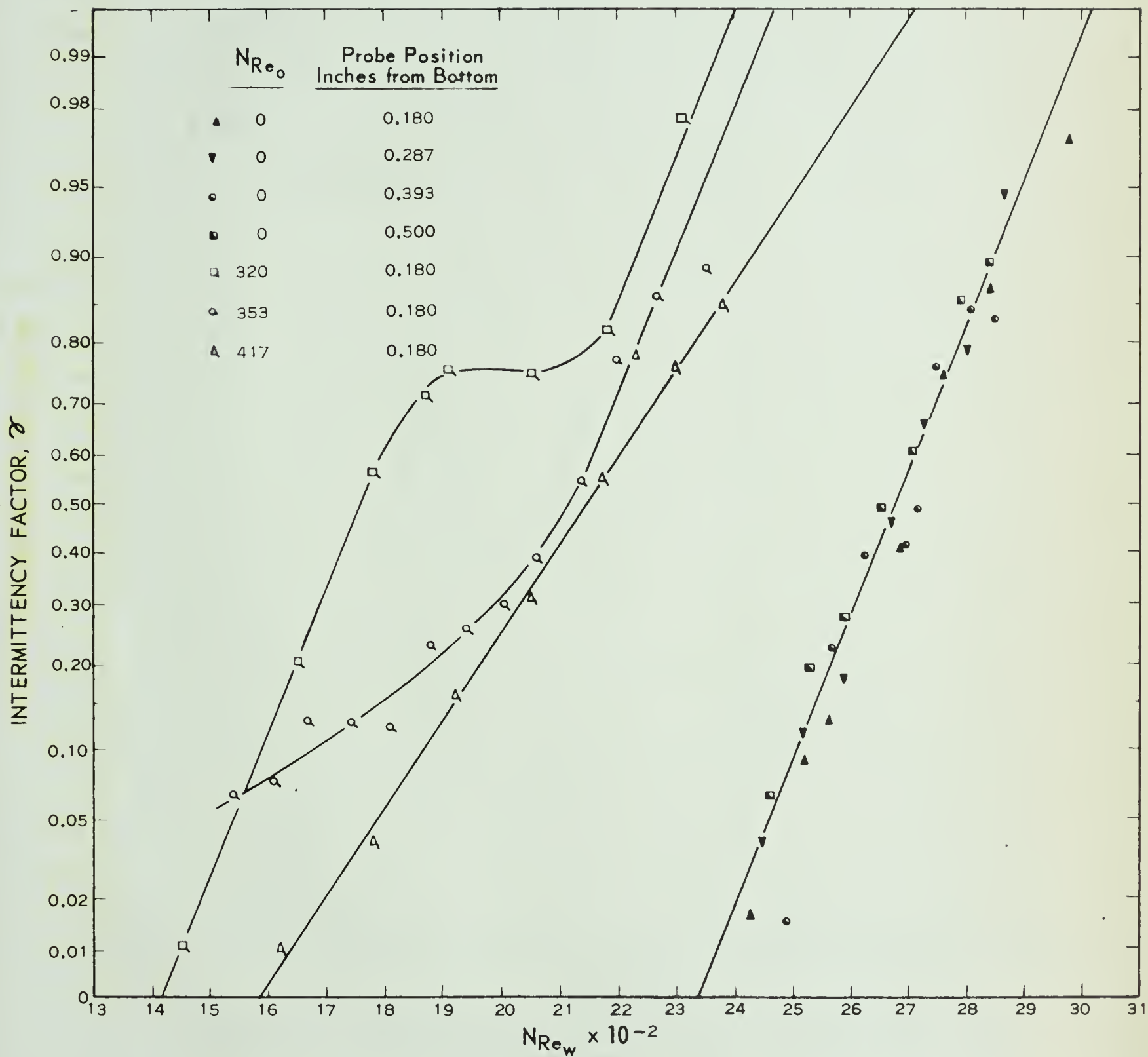


FIGURE 24. Probability plot of intermittency factor versus water Reynolds number
Oil Reynolds numbers 0 and 320 to 417

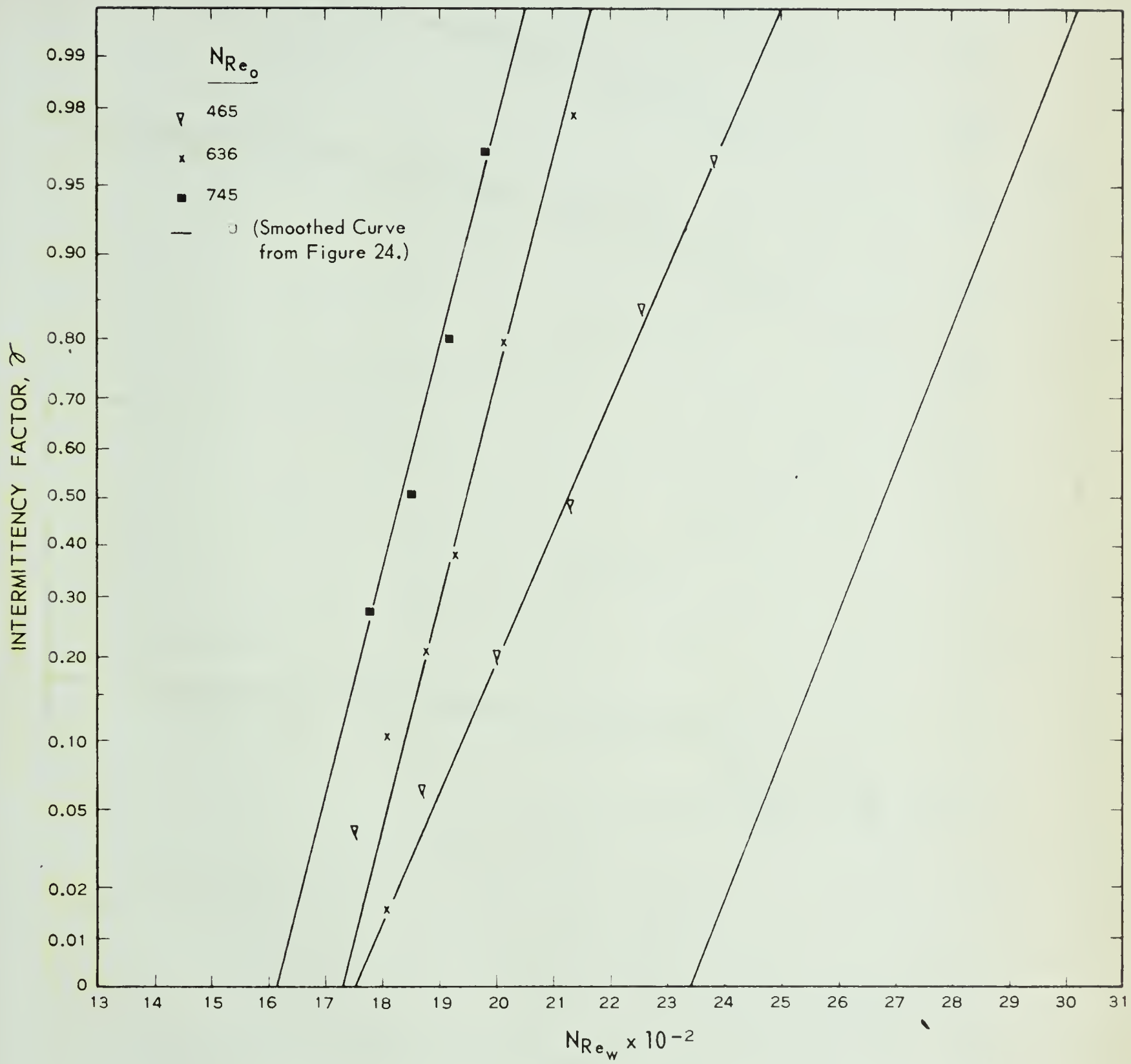


FIGURE 25. Probability plot of intermittency factor versus water Reynolds number
Oil Reynolds numbers 0 and 465 to 745

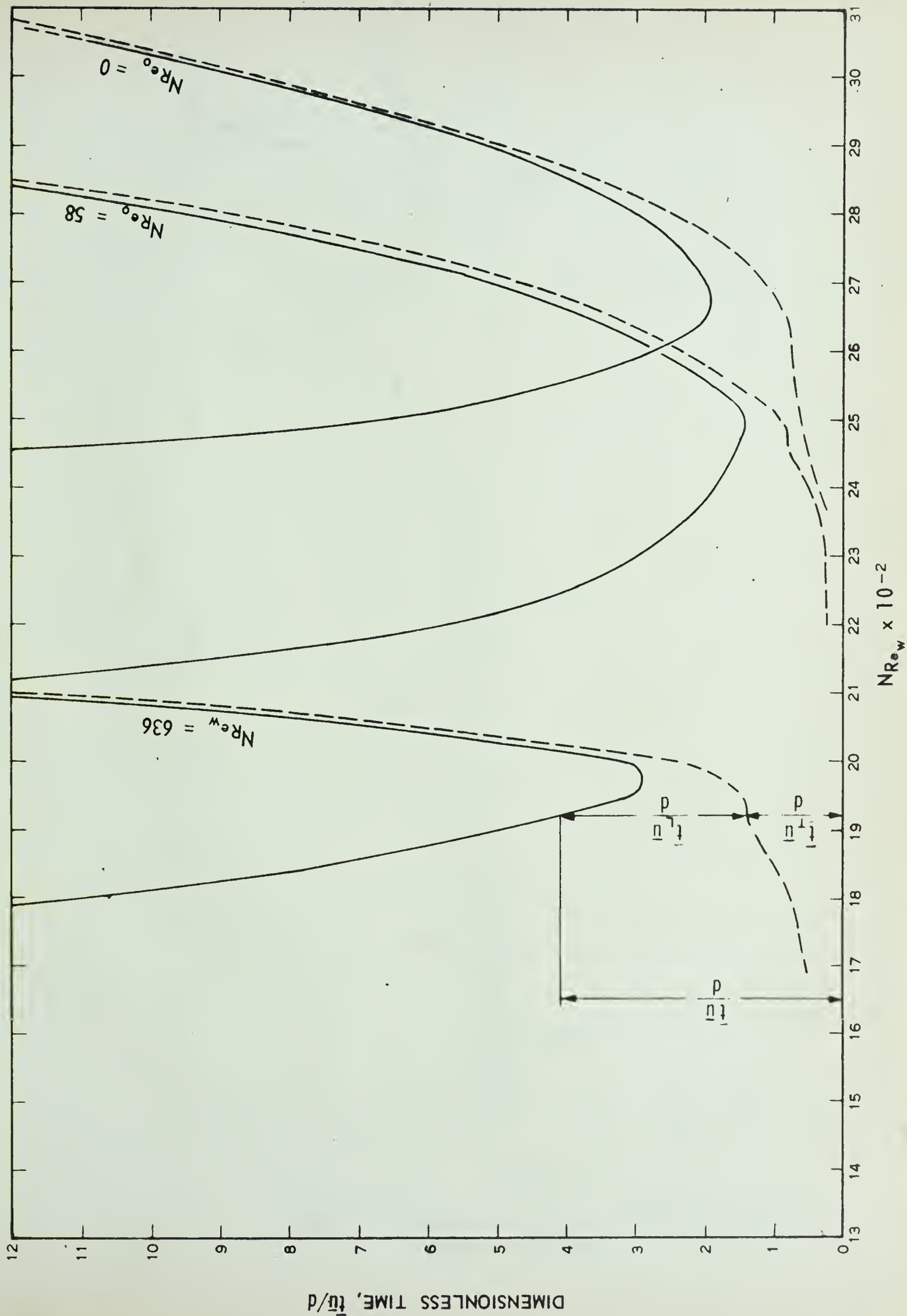


FIGURE 26. Average time of laminar-turbulent flow cycle as a function of water Reynolds number for oil Reynolds number 0, 58 and 636

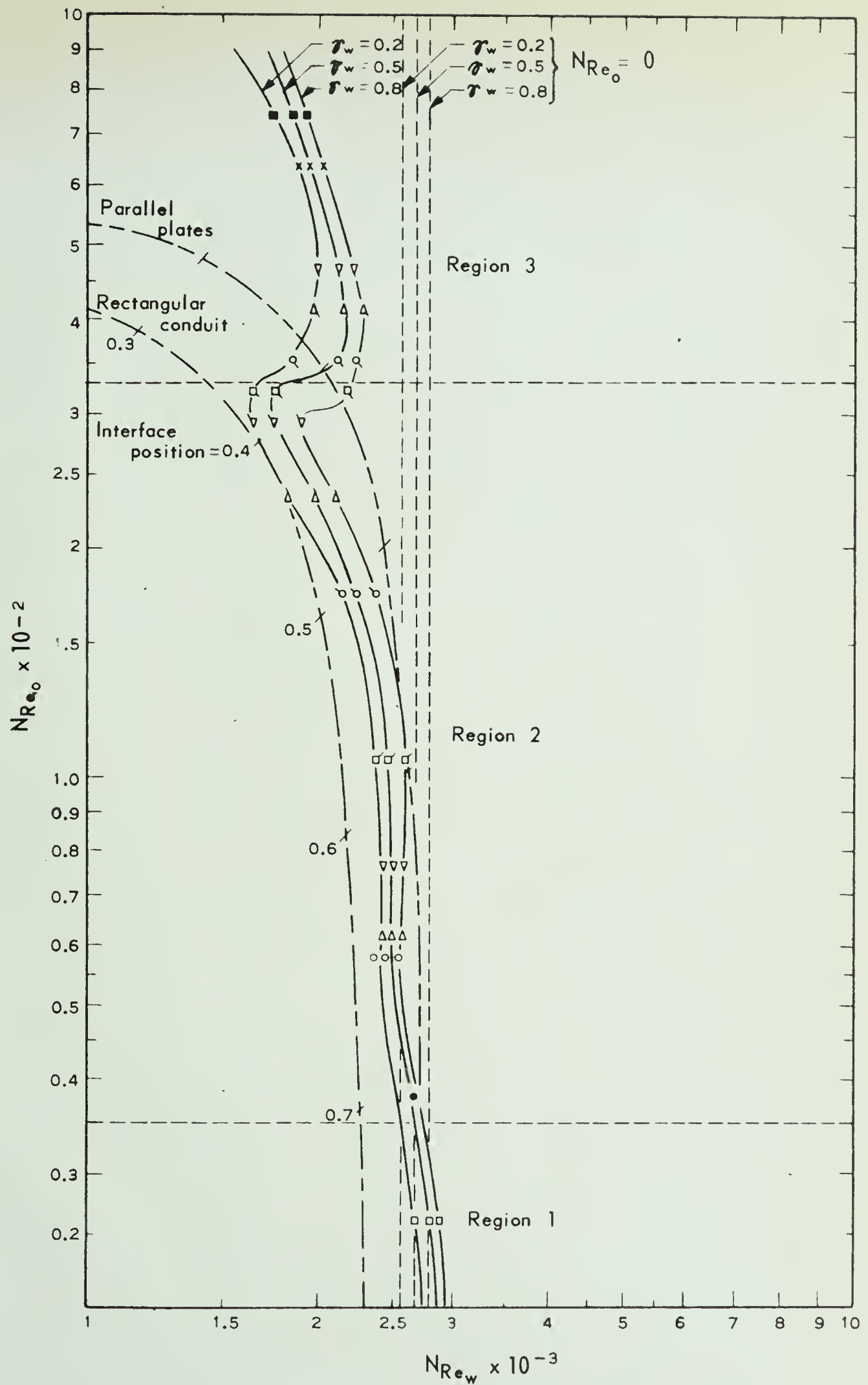


FIGURE 27. Oil Reynolds number as a function of transition water Reynolds number

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